TALK

From the birth to the *death of stars*



From a cold cloud of hydrogen to a white dwarf, a neutron star or a black hole — and the physics that decides *which* end it will be.

$M_{\mathrm{J}} \cdot M_{\mathrm{Ch}} \cdot M_{\mathrm{TOV}}$ THREE MASSES DECIDE THE FATE

Rafael C. R. de Lima

Department of Physics · UDESC/CCT · Joinville, Brazil

rafael-lima.pages.dev/talks

OUTLINE

Five parts, *one life*

I

The birth

Molecular clouds, the Jeans criterion and why a shrinking cloud ends up lighting up.

II

The life

Fusion against gravity, mass and lifetime, the H–R diagram and what happens when the hydrogen runs out.

III

Degenerate stars

Heisenberg and Pauli hold up what fusion no longer can — up to the Chandrasekhar limit.

IV

Neutron stars

Pulsars, magnetars, nuclear density and light that bends — with results from my own research.

V

Black holes

From Laplace to Schwarzschild, the horizon, time that freezes and the stars around Sgr A*.

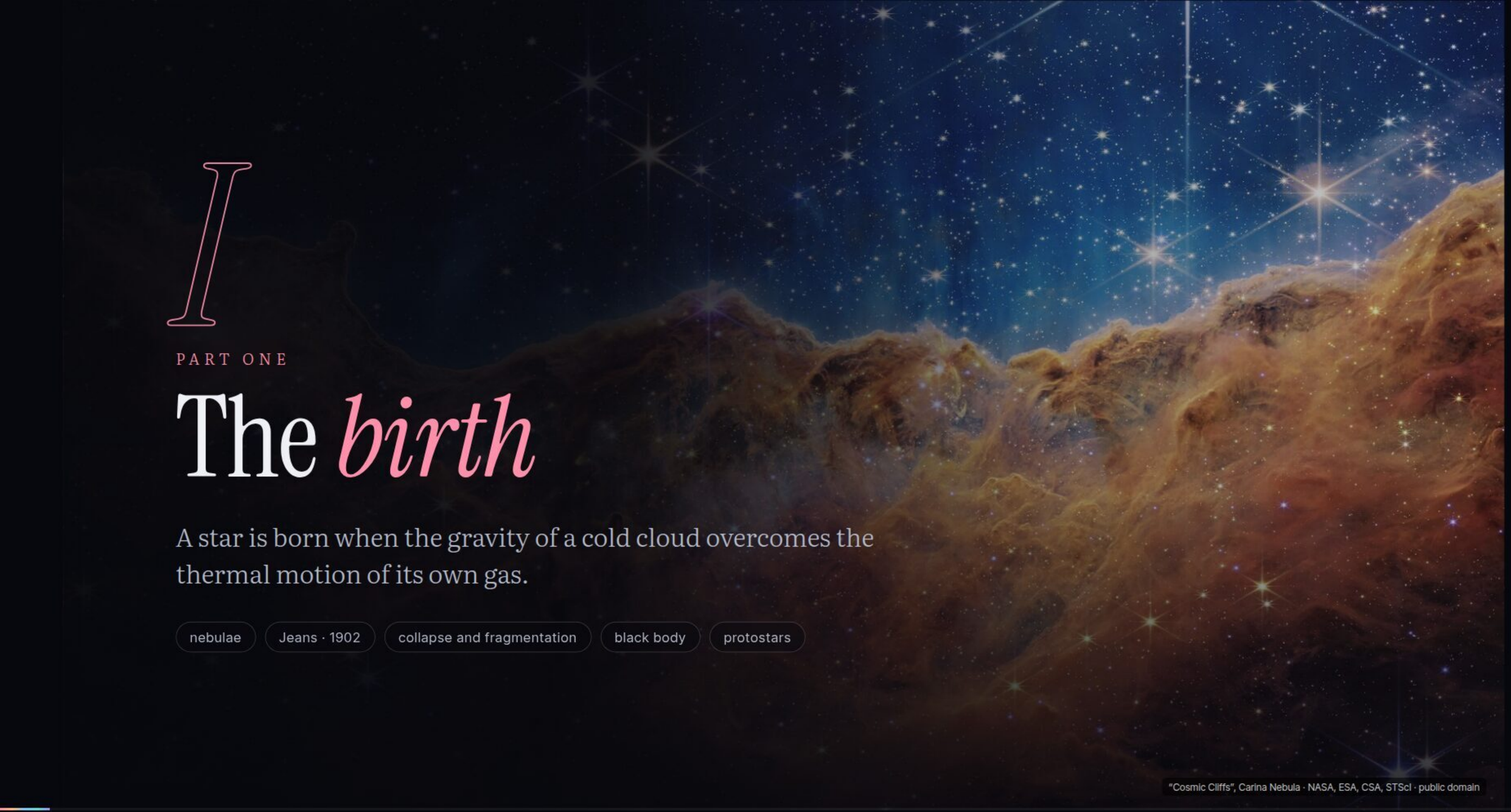
I

PART ONE

The *birth*

A star is born when the gravity of a cold cloud overcomes the thermal motion of its own gas.

nebulae | Jeans · 1902 | collapse and fragmentation | black body | protostars



"Cosmic Cliffs", Carina Nebula · NASA, ESA, CSA, STScI · public domain

MOLECULAR CLOUDS

Where is *a star born?*



In nebulae: cold clouds of gas and dust.

- **Molecular clouds**: the hydrogen is in molecules, H_2.
- **Dense enough for chemistry**: H_2, carbon monoxide (CO), formaldehyde (H_2CO), water, alcohol…
- **Cold**: 10 to 20 K, around −260 °C.

$10–20$ K	$10^2–10^6$	10^6 $M_\odot$
temperature	molecules per cm^3	up to this, in a giant cloud

Even the densest part is a better vacuum than any laboratory's: the air we breathe has 2.5×10^{19} molecules per cm^3.

EAGLE NEBULA · M16 · 7,000 LIGHT-YEARS

The *Pillars of Creation*





Stars are being born in the pillars: the **protostars**.

In visible light the dust hides them. In the infrared it becomes transparent, and they show up as reddish dots at the tips of the pillars.

The left-hand pillar is about 4 light-years tall. Hubble: 1995, retaken in 2014. Webb: 2022.

SIR JAMES JEANS · 1902

When does gravity *beat heat?*

THE IDEA

A region of radius R and mass M collapses if its molecules, stirred by the cloud's temperature T, **are not fast enough to escape** from it.

Then gravity beats the pressure that comes from thermal energy, and any disturbance makes the region fall in on itself.

It is the same escape-velocity sum that will come back in Part V — there, with light in place of the molecules.

Mean kinetic energy of a molecule of mass m, and its gravitational potential energy at the edge of the region:

$$K_{\text{avg}} = \frac{3}{2} k_{\text{B}} T \qquad U_{\text{G}} = \frac{GMm}{R}$$

The threshold for collapse is where the two are equal:

$$\frac{3}{2} k_{\text{B}} T = \frac{GMm}{R}, \quad M = \frac{4\pi}{3} \rho R^3$$

Solving for R:

$$R = \frac{9\, k_{\text{B}} T}{8\pi G \rho m}^{1/2}$$

The denser the cloud, *the smaller the piece* that collapses

In a more careful analysis, with pressure perturbations, Jeans found:

$$\lambda_J = \frac{\pi k_B T}{G\rho m}^{1/2}$$

JEANS LENGTH

$$M_J = \frac{4\pi}{3}\lambda_J^3 \rho = \frac{4\pi^{5/2}}{3}\frac{k_B T}{Gm}^{3/2} \rho^{-1/2}$$

JEANS MASS

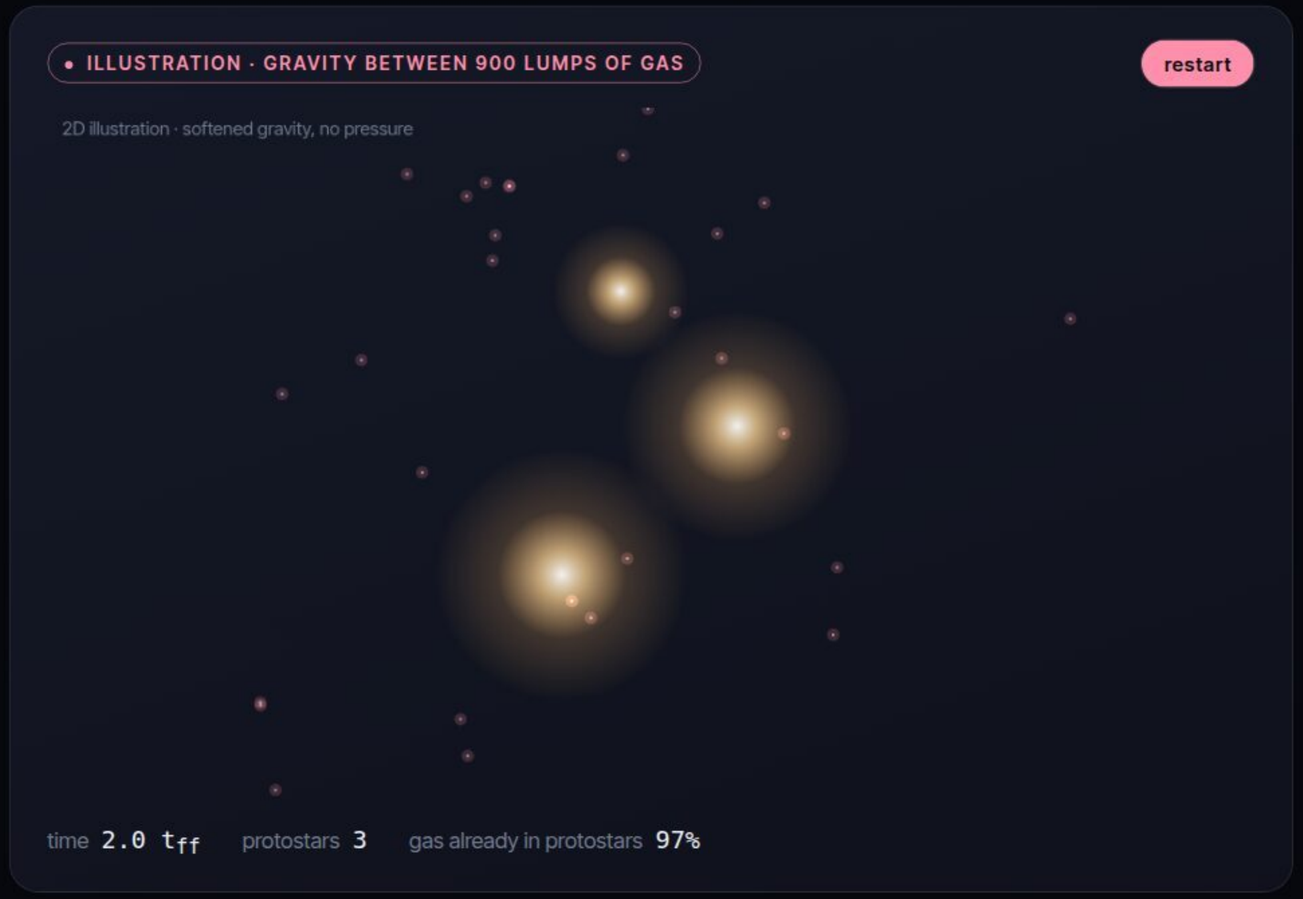
$$M_J \propto T^{5/2} \rho^{-1/2}$$

As the cloud collapses and gets denser, the mass that can collapse on its own **decreases**: the cloud breaks up into pieces — and each piece becomes a star. That is why stars are born in groups.



COLLAPSE AND FRAGMENTATION

From the cloud to *protostars*



The cloud falls in on itself. The densest regions collapse fastest — the free-fall time goes as $\rho^{-1/2}$ — and the collapse **fragments** the cloud into several cores.

Each fragment spins faster and faster as it shrinks, like a skater pulling in her arms: it forms a **disc**, with a **protostar** at its centre.

Illustration, not simulation: particles with softened gravity and a little thermal motion, in a plane. Real gas has pressure, turbulence, magnetic fields and radiation — full simulations in Krumholz, Klein & McKee, *ApJ* 656, 959 (2007).

THE DIFFERENCE IS TEMPERATURE

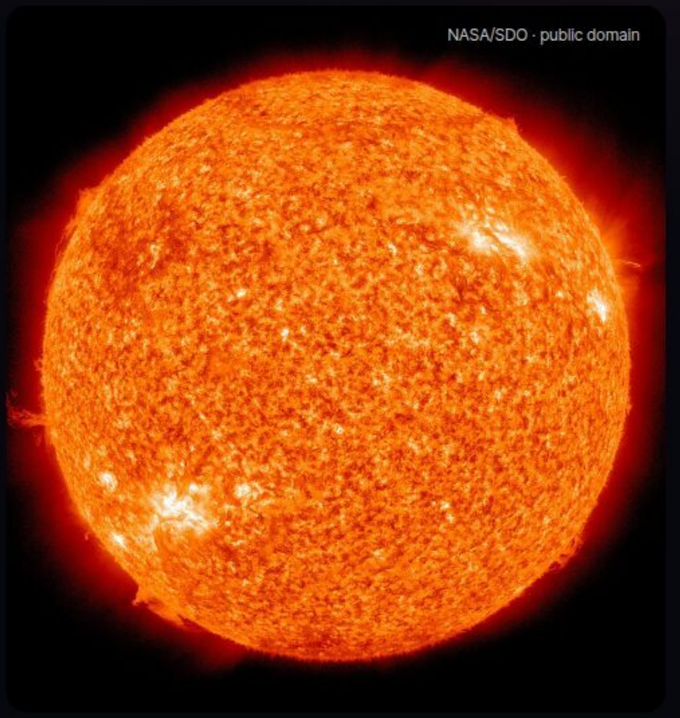
What is the difference between a cloud *and a star?*



Cloud · 10 K · shines in radio and infrared



Protostar with a disc · HL Tauri · ~1 million years



Star · the Sun · 5,772 K at the surface, 15 million K at the centre

contraction → rising temperature → black body → visible light → **a star is born!**

BLACK-BODY RADIATION · PLANCK 1900

A star is a black body… *almost*

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_B T} - 1}$$

PLANCK'S LAW

$$\lambda_{max} = \frac{2.898\ \text{mm·K}}{T}$$

WIEN

$$L = 4\pi R^2 \sigma T^4$$

STEFAN–BOLTZMANN

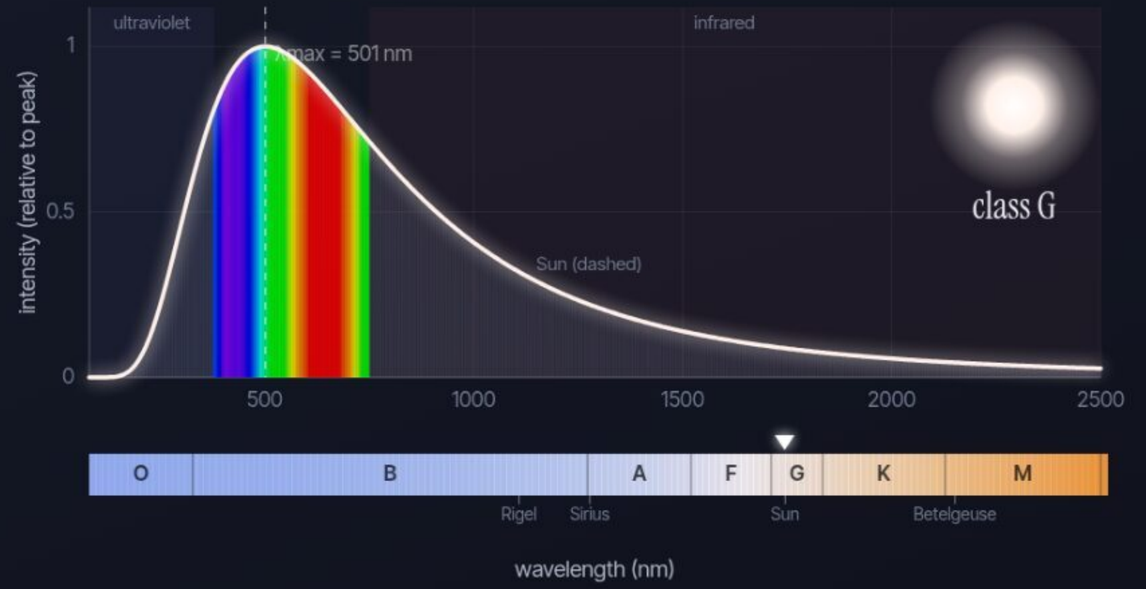
The hotter the star, the bluer the peak and the far greater the brightness: doubling T multiplies the luminosity by 16.

The spectral classes **O B A F G K M** are a temperature scale: from over 30,000 K (blue) to under 3,700 K (red). The Sun is a G2.

"Almost": the real spectrum has absorption lines from the elements in the atmosphere, and that is how we know what stars are made of.

BLACK-BODY SPECTRUM · COLOUR OF THE STAR

drag the temperature

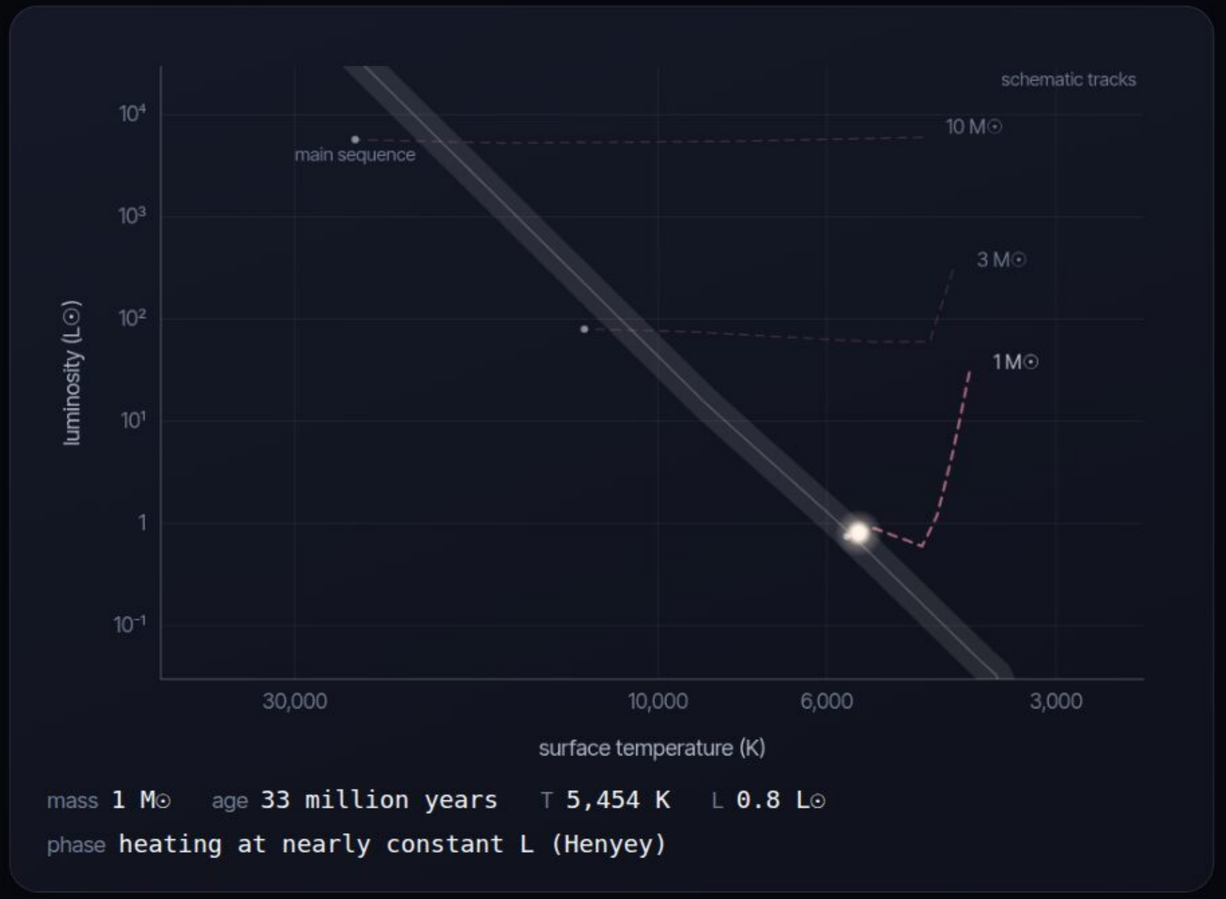


surface temperature 5,781 K

λ_{max} 501 nm in the visible class G brightness per m², relative to the Sun 1.01×

HAYASHI · HENYEY · 1961

From protostar to *main sequence*



- The protostar starts as an **invisible** concentration of gas deep inside the cloud.
- It contracts and heats up, hidden in a **cocoon of dust**: it only shows up in the infrared.
- It blows the cocoon away and becomes **visible**, still shrinking: first almost straight down the diagram (the Hayashi track), then to the left (the Henyey track).
- When the centre reaches ~10 million K, hydrogen starts to fuse: the star joins the **main sequence** and stops shrinking.

Schematic tracks, with key points roughly following Iben's models (1965). The more massive the star, the faster: ~50 million years for 1 M☉, ~1 million for 5 M☉.

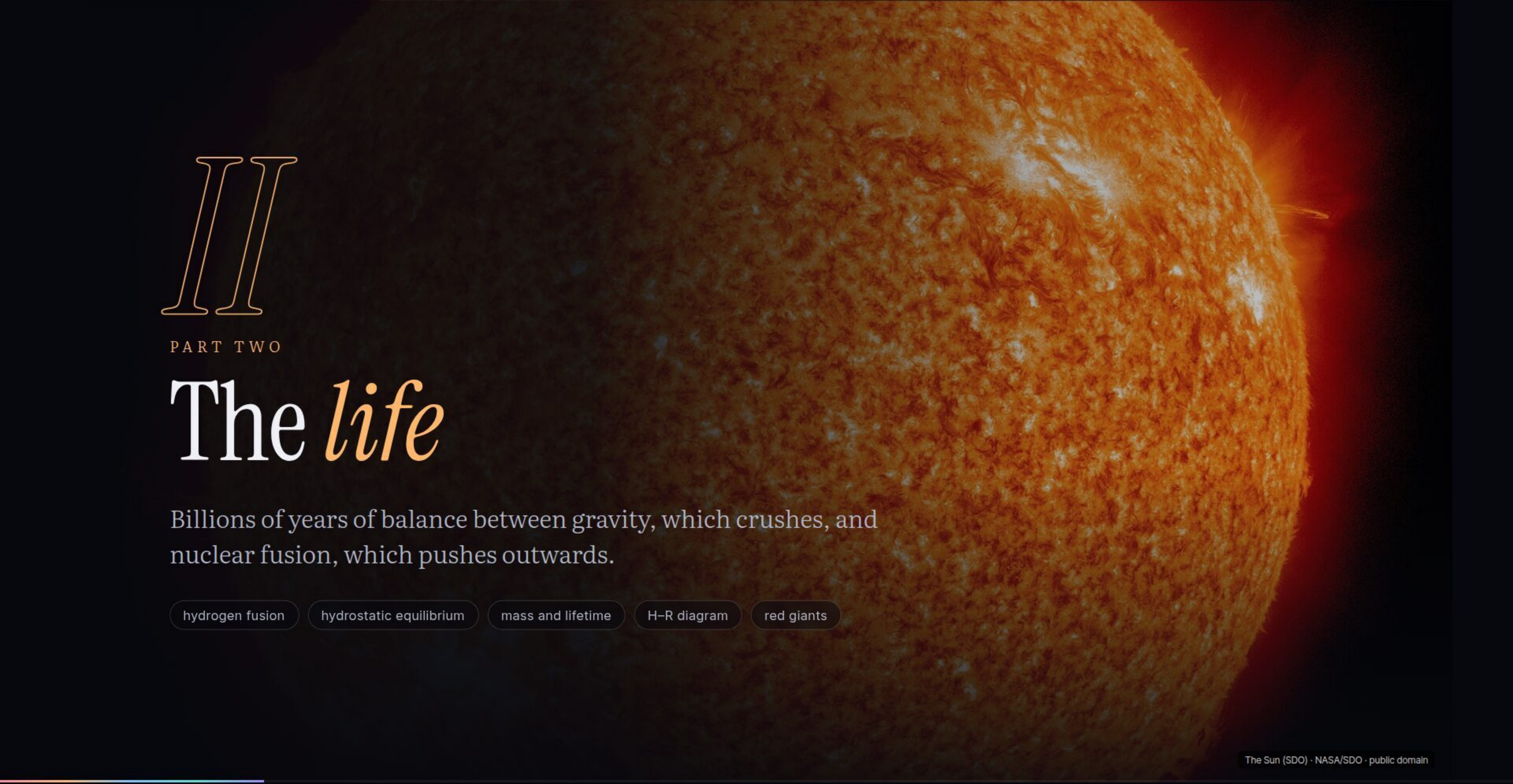
II

PART TWO

The *life*

Billions of years of balance between gravity, which crushes, and nuclear fusion, which pushes outwards.

hydrogen fusion · hydrostatic equilibrium · mass and lifetime · H–R diagram · red giants

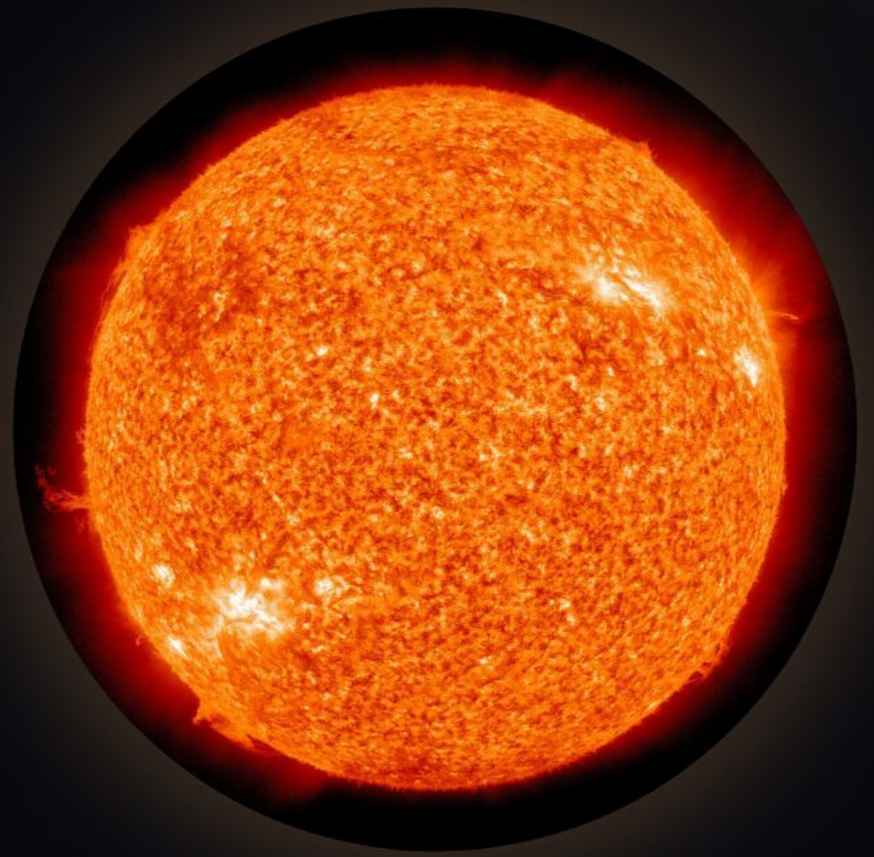


The Sun (SDO) · NASA/SDO · public domain

Brightness, stability... *and peace?*

Brightness.
Stability.
~~Peace.~~

Not peace. At the centre of every star, non-stop, the reaction of a *hydrogen bomb* is going on.



PROTON–PROTON CHAIN

The heart of a star is *(literally)* a hydrogen bomb



NNSA / Nevada Site Office · public domain

Ivy Mike, the first hydrogen bomb (1952): 10.4 megatons

$$^1H + {}^1H \rightarrow {}^2H + e^+ + \nu_e$$

$$^2H + {}^1H \rightarrow {}^3He + \gamma$$

$$^3He + {}^3He \rightarrow {}^4He + 2\,{}^1H$$

IN TOTAL: $4\ {}^1H \rightarrow {}^4HE + 2\ E^+ + 2\ N_E + 26.7$ MEV

NUCLEAR REACTIONS KEEP UP

- the temperature — the black-body glow
- the pressure that prevents gravitational collapse

GRAVITY KEEPS

- the star together — the bomb does not fly apart
- radiation trapped: a photon takes ~100,000 years to get out

610 million t/s

of hydrogen turn into helium

4.3 million t/s

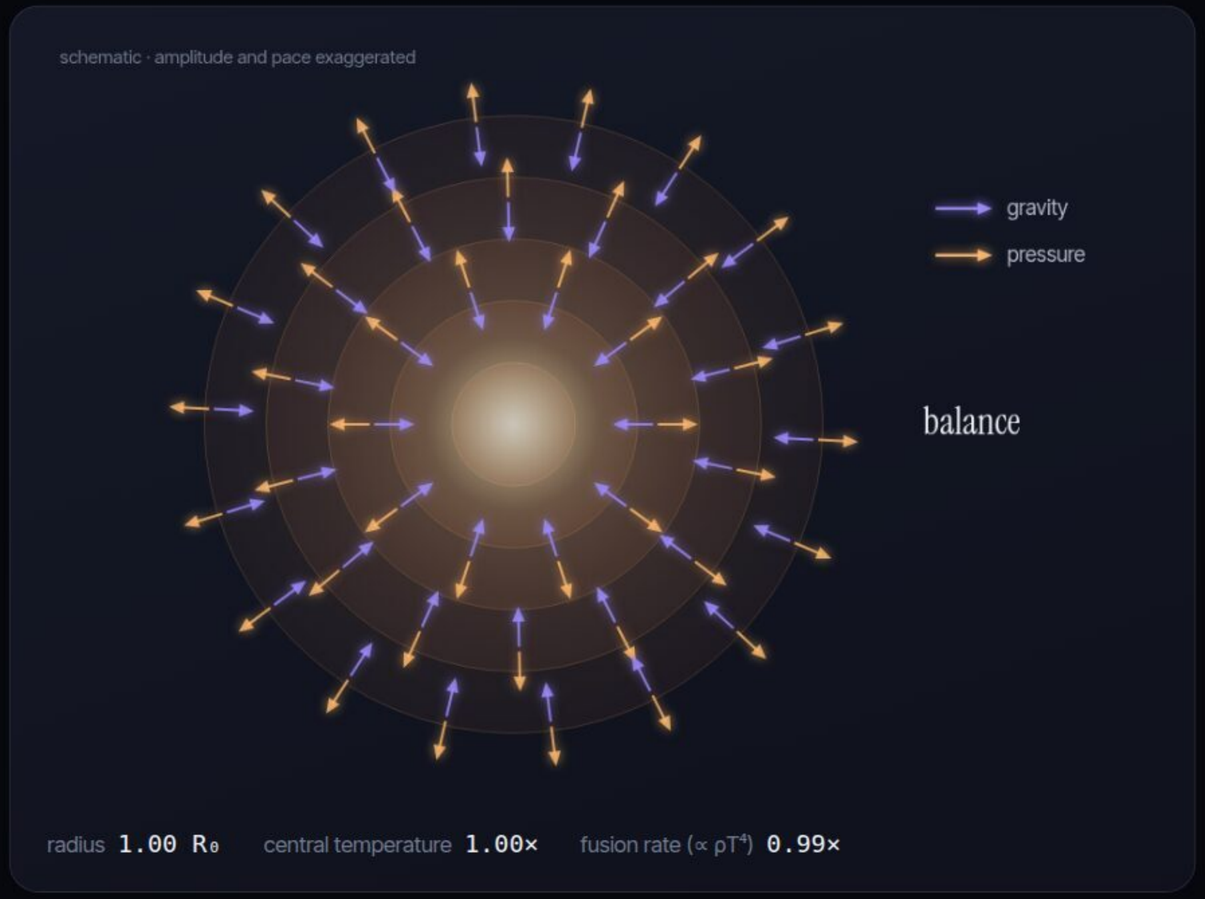
of mass turn into light ($E = mc^2$)

9×10^9

Ivy Mikes per second

HYDROSTATIC EQUILIBRIUM

A *violent* balance



$$\frac{dP}{dr} = -\frac{G\,m(r)\,\rho(r)}{r^2}$$

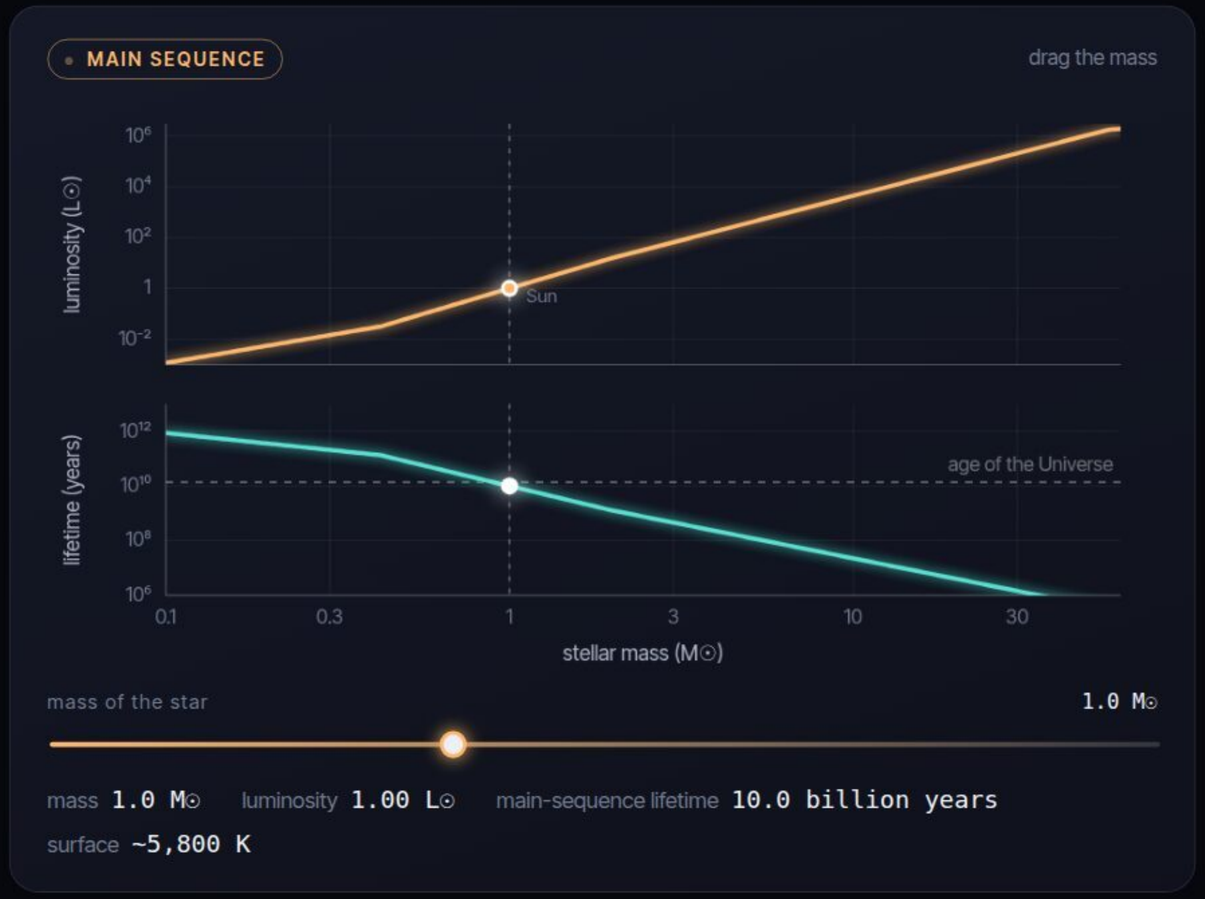
IN EVERY LAYER: PRESSURE AGAINST WEIGHT

In every layer, the weight of everything above is held up by the difference in pressure. If the star shrinks, the centre heats up, fusion speeds up and the pressure pushes back; if it swells, it cools and fusion slows down. **A thermostat.**

CENTRE OF THE SUN	VALUE
temperature	15.7 million K
density	150 g/cm³
pressure	2.5×10¹⁶ Pa

Without pressure, the Sun would collapse in ~30 minutes — the free-fall time at its mean density.

More mass, *shorter life*



$$L \propto M^{3.5}$$

MASS–LUMINOSITY RELATION (2 TO 55 M⊙)

$$t \approx 10^{10}\ \text{years} \cdot \frac{M/\text{M}\odot}{L/\text{L}\odot} \propto M^{-2.5}$$

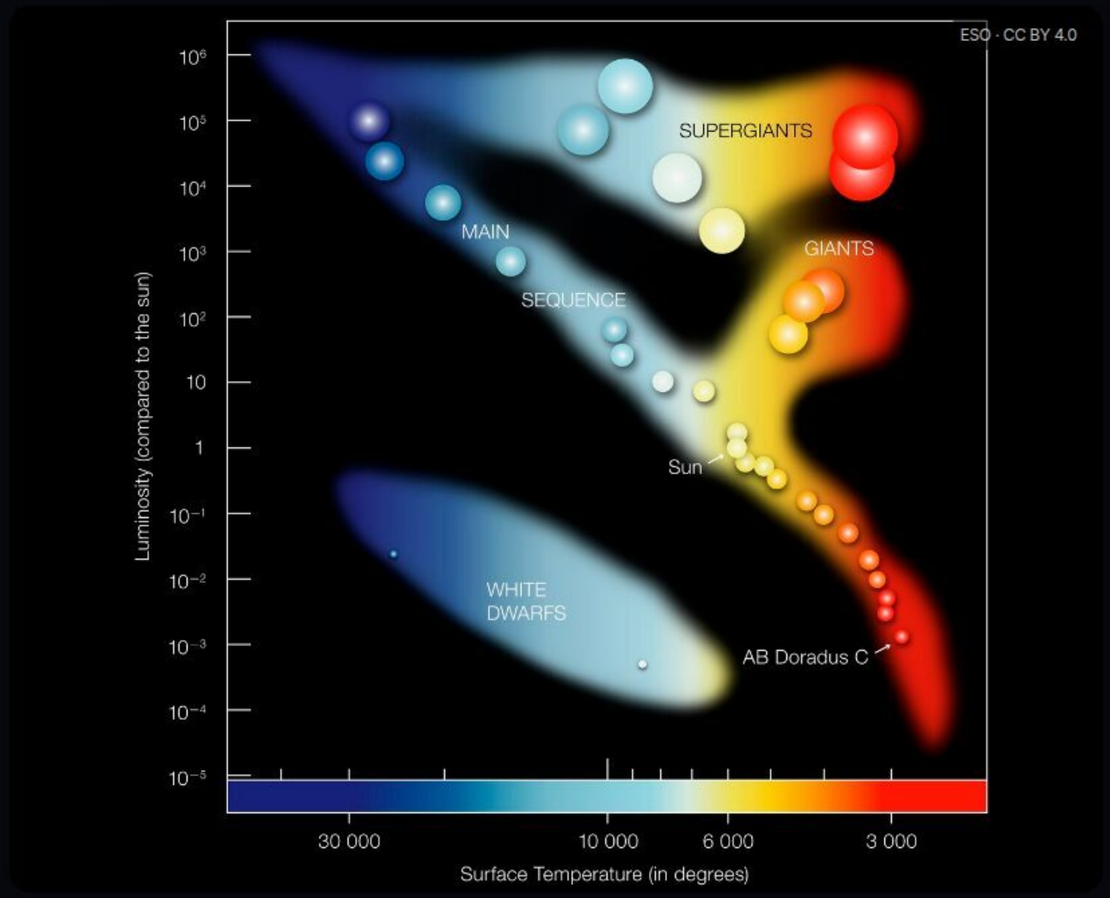
FUEL ÷ BURNING RATE

A 10 M⊙ star has ten times more fuel, but shines thousands of times brighter: it lives **~30 million years**. The Sun, ~10 billion. A red dwarf of 0.2 M⊙, trillions — longer than the age of the Universe.

Every low-mass star ever born is still alive.

HERTZSPRUNG · 1911 · RUSSELL · 1913

The *H–R* diagram



Temperature (or colour) on the horizontal axis, increasing to the **left**; luminosity on the vertical axis.

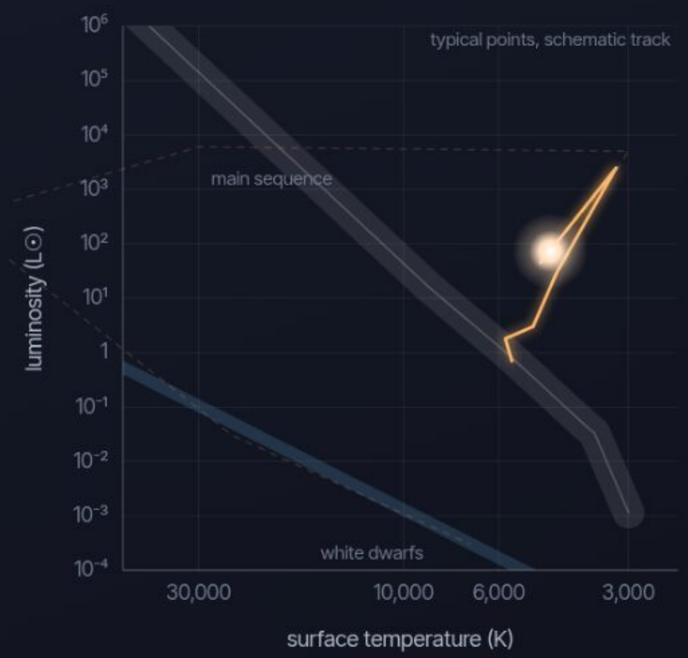
- **Main sequence**: the diagonal, where stars spend 90% of their lives burning hydrogen in the core.
- **Giants and supergiants**: cool and huge, top right.
- **White dwarfs**: hot and small, bottom left.

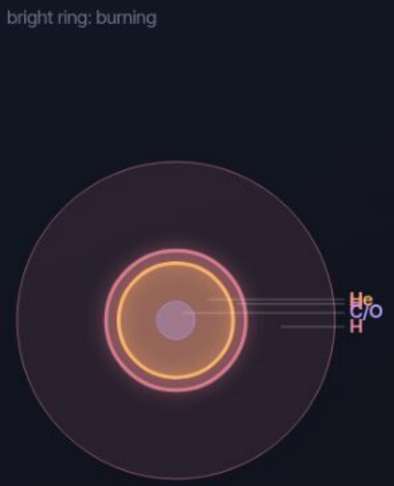
When the star runs out of hydrogen in its core, little energy is left to support the weight of the layers above — **90% of the star**.

Three things *at once*

• EVOLUTION ON THE H–R DIAGRAM · SCHEMATIC

1 M⊙ · the Sun | 15 M⊙





phase horizontal branch: He → C, O in the core duration ~100 million years

T 4,600 K L 72 L⊙ R 14 R⊙

1 · GRAVITY STARTS TO WIN

The helium core contracts, and the contraction relights hydrogen burning in a **shell** around it.

2 · THE CORE HEATS UP

Until another reaction ignites: $^{4}He \rightarrow {}^{12}C$ (triple-alpha, at 100 million K); in massive stars, ${}^{12}C \rightarrow {}^{16}O \rightarrow \ldots \rightarrow {}^{56}Fe$, in shells like those of an onion.

3 · THE OUTER LAYERS SWELL

The energy pushes the envelope out: the star grows and cools — a **red giant**. Above ~8 M⊙, a **red supergiant**.

AND WHEN THE FIRE GOES OUT?

Two paths, *three fates*



Molecular cloud

10 K · millions of M☉

M ≲ 8 M☉



Sun-like star

billions of years



Red giant

~100 R☉



Planetary nebula

the envelope drifts away



White dwarf

the size of the Earth

M ≳ 8 M☉



Massive star

millions of years



Red supergiant

~1,000 R☉ · Betelgeuse



Supernova

core collapse



Neutron star or black hole

above ~20–25 M☉: black hole

GRAVITATIONAL COLLAPSE

When the fuel *runs out*

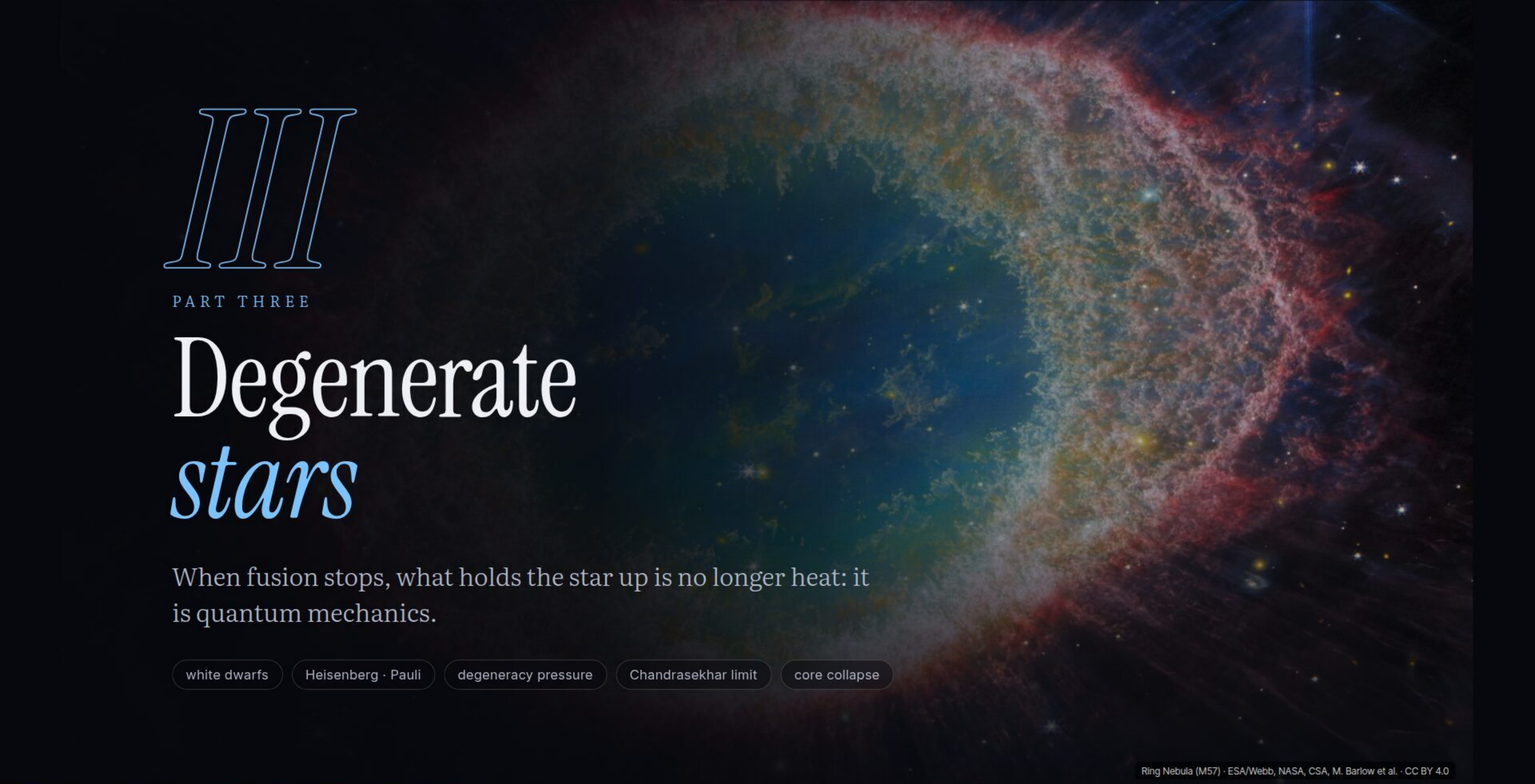
Without nuclear reactions, the pressure they kept up is gone: gravity wins and the star's matter goes into *free fall*.

For some stars there is still a way out: *degeneracy pressure*.

HOW LONG DOES IT TAKE?

$$t_{\mathrm{ff}} = \frac{3\pi}{32\, G\, \rho}^{1/2}$$

At the Sun's mean density, ~30 minutes. For the iron core of a massive star, ~0.1 s.



III

PART THREE

Degenerate *stars*

When fusion stops, what holds the star up is no longer heat: it is quantum mechanics.

white dwarfs | Heisenberg · Pauli | degeneracy pressure | Chandrasekhar limit | core collapse

Ring Nebula (M57) · ESA/Webb, NASA, CSA, M. Barlow et al. · CC BY 4.0

The size of the Earth, or *of a city*

WHITE DWARFS



Sirius B · R ≈ 5,600 km

Earth · R = 6,371 km

$R \approx R_\oplus$ · $M \approx M_\odot$ · $\rho \approx 10^6$ g/cm³

A teaspoonful weighs ~5 tonnes. Held up by **degenerate electrons**.

NEUTRON STARS



$R \approx 12$ km · $M \approx 1.4$–$2\ M_\odot$ · $\rho \approx 10^{14}$–10^{15} g/cm³

A teaspoonful weighs ~500 million tonnes. Held up by **degenerate neutrons** and the nuclear force.

QUANTUM MECHANICS · 1925 – 1927

Heisenberg and Pauli *to the rescue*



Werner Heisenberg
1901 – 1976



Wolfgang Pauli
1900 – 1958

UNCERTAINTY PRINCIPLE · 1927

$$\Delta p \, \Delta x \geq \frac{\hbar}{2}$$

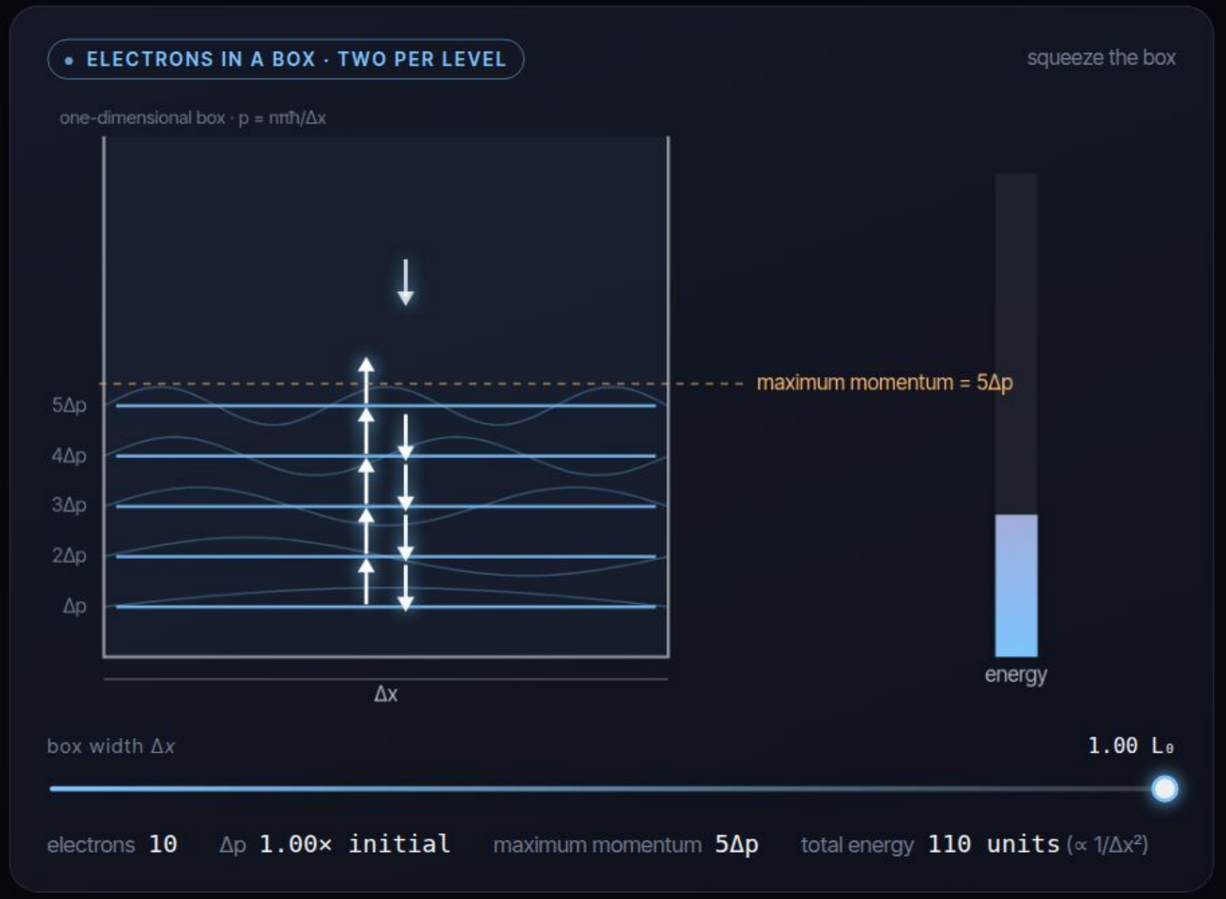
Confining a particle to a small region forces its momentum to be large.

EXCLUSION PRINCIPLE · 1925

Two identical fermions — electrons, protons, neutrons — **cannot occupy the same quantum state**.

Together: squeezing fermions costs energy, **even at zero temperature**.

Why fermions *push back*



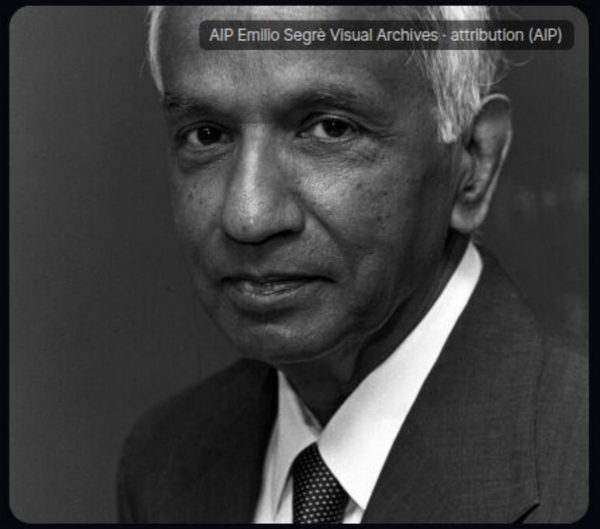
Each energy level holds **two** electrons: one with spin up, the other spin down. The next ones are forced to go higher.

In a box of width Δx, the levels are spaced by $\Delta p \approx \hbar/\Delta x$. With N electrons, the last one has momentum $N/2 \cdot \Delta p$: if $N_e = 6$, the maximum momentum is $\mathbf{3\Delta p}$.

Squeezing the box pushes the levels apart: the electrons gain momentum with no need for heat. That is **degeneracy pressure**.

CHANDRASEKHAR · 1930 · NOBEL PRIZE 1983

The maximum mass *of white dwarfs*



S. Chandrasekhar

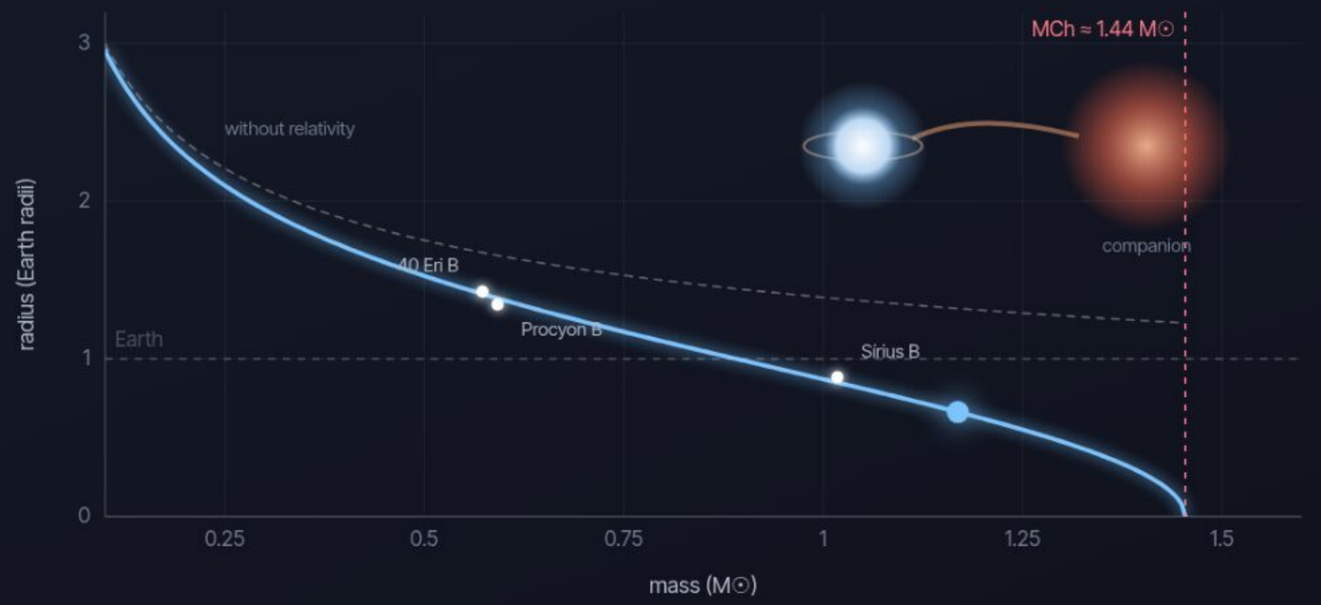
1910 – 1995

$$M_{Ch} = 3.10 \frac{(\hbar c/G)^{3/2}}{(\mu_e m_H)^2} \approx 1.44\,M\odot$$

M_E = 2 · CARBON AND OXYGEN

MASS–RADIUS RELATION OF WHITE DWARFS

the white dwarf gains mass from a companion



mass 1.169 M⊙ radius 4,225 km mean density 7.4×10^6 g/cm³ fraction of M_{Ch} 80%

Curve: Nauenberg's approximation (1972) for an electron gas at any degree of relativity. Points: Sirius B, Procyon B and 40 Eridani B. The more mass, the **smaller** the white dwarf — and at M_{Ch} the radius goes to zero.

Collapse! *Next stop: neutron stars*



core collapse · supernova · remnant · pulsar + nebula

At the end of a massive star's life, the iron core passes 1.4 M☉ and the electrons can no longer hold it up. In less than a second it falls from ~3,000 km to ~30 km, the electrons are captured by the protons, turning them into neutrons — and the rest of the star is blown off in a supernova. A neutron star is left at the centre.

Artist's animation · NASA's Goddard Space Flight Center Conceptual Image Lab (animation: Jonathan North) · SVS 20413 · public domain. A white dwarf that crosses the limit by accreting matter also explodes — a type Ia supernova — but leaves nothing behind.

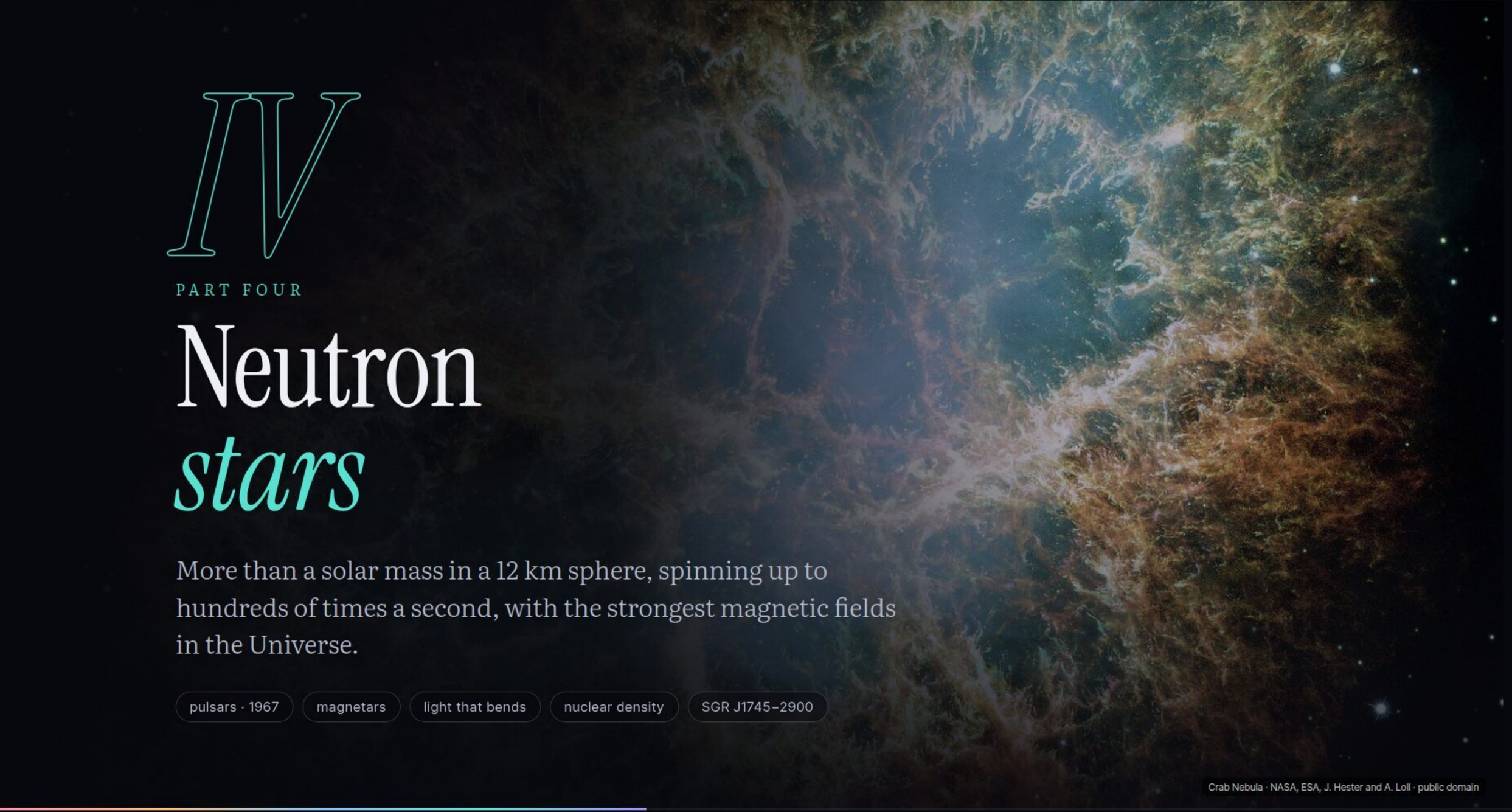
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PART FOUR

Neutron *stars*

More than a solar mass in a 12 km sphere, spinning up to hundreds of times a second, with the strongest magnetic fields in the Universe.

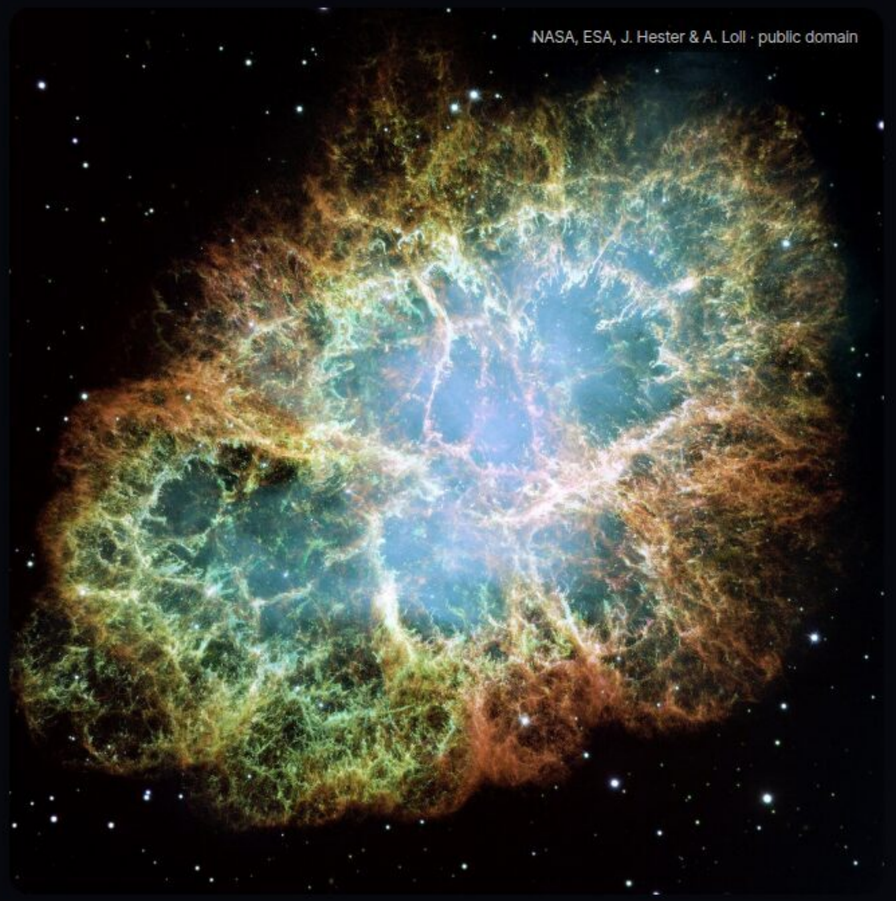
pulsars · 1967 | magnetars | light that bends | nuclear density | SGR J1745−2900



Crab Nebula · NASA, ESA, J. Hester and A. Loll · public domain

SUPERNOVA OF 1054 · 6,500 LIGHT-YEARS

The *Crab Nebula*



1054

recorded by Chinese astronomers: visible in daylight for 23 days

33 ms

period of the pulsar at the centre: 30 turns a second

4×10^{38} erg/s

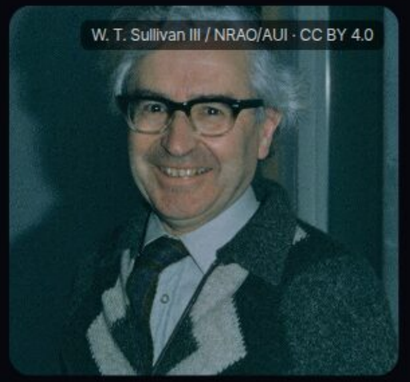
lost by the rotation as it slows down — this is what lights up the whole nebula

CAMBRIDGE · 28 NOVEMBER 1967

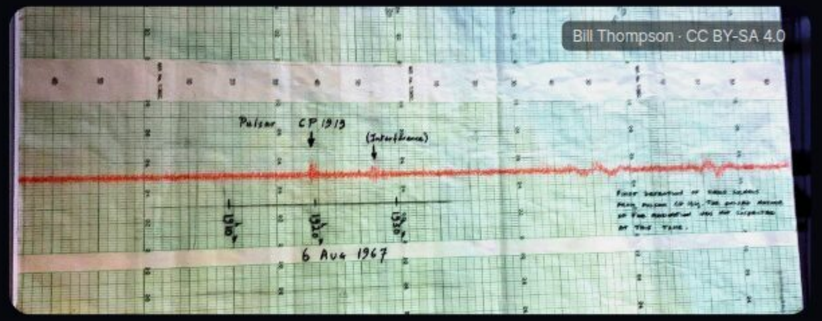
The first pulsar: *“LGM-1”*



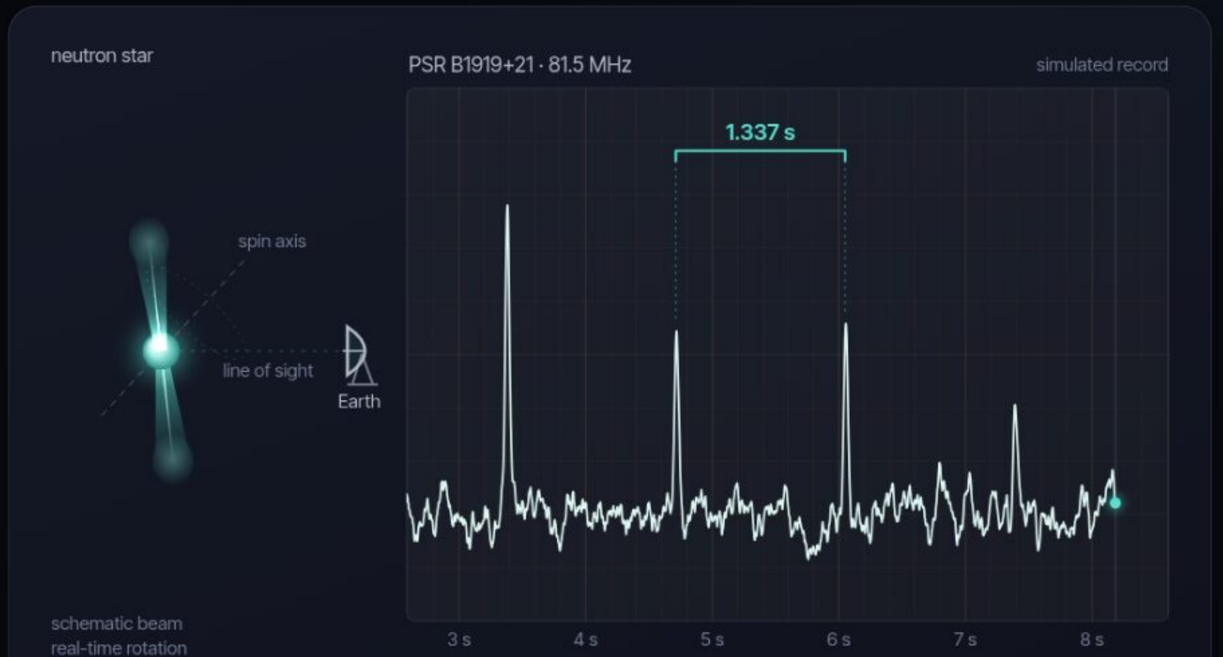
Jocelyn Bell
in 1967, PhD student



Antony Hewish
supervisor · Nobel Prize 1974



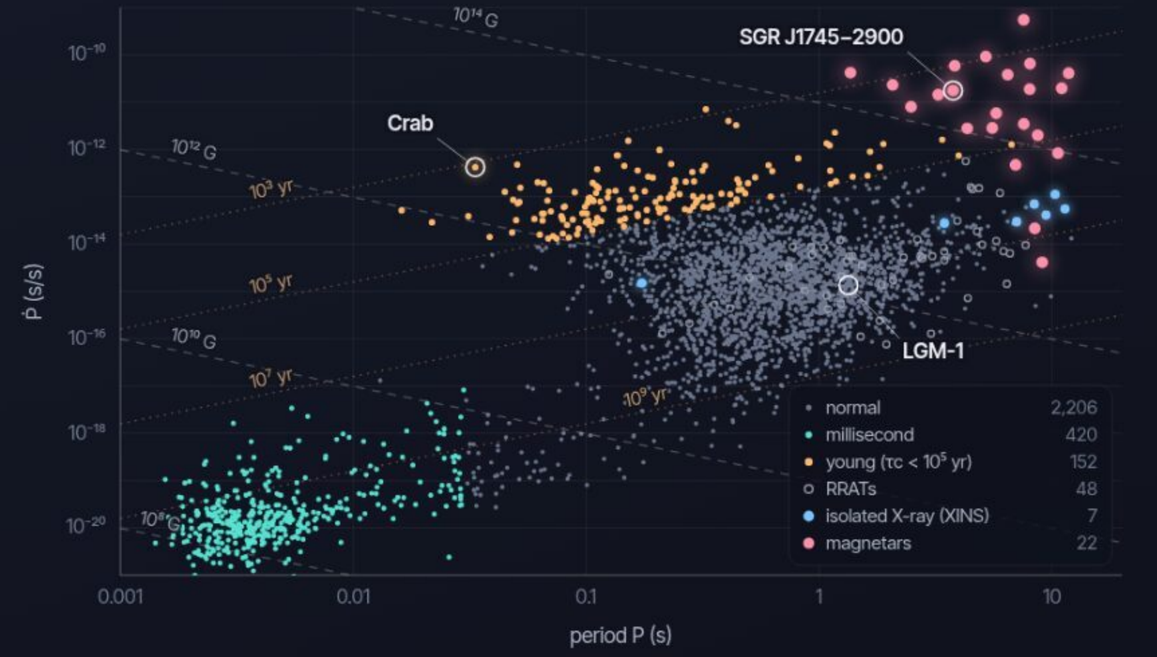
The original chart, August 1967: the “scruff” pencilled in as CP 1919.



period `1.337 302 s` Ṗ `1.35×10⁻¹⁵ s/s` frequency `81.5 MHz` pulses recorded `6`

Bell noticed a bit of “scruff” that kept coming back on the radio telescope’s paper chart: pulses every **1.337 s**, as regular as clockwork. Nickname: **LGM-1**, for *Little Green Men*. It was a spinning neutron star.

What is measured, *and what is inferred*



OBSERVABLE	EXPRESSION
period	P
characteristic age	$t_c = P/2\dot{P}$
magnetic field	$B = (3Ic^3P\dot{P}/8\pi^2R^6)^{1/2}$
rotational energy lost	$\dot{E}_{rot} = -4\pi^2I\dot{P}/P^3$

CANONICAL NEUTRON STAR

$M = 1.4\ M\odot \cdot R = 10\ km \cdot I = 10^{45}\ g\ cm^2$

$B = 3.2\times10^{19}\ (P\dot{P})^{1/2}\ G$

$|\dot{E}_{rot}| = 3.9\times10^{46}\ \dot{P}/P^3\ erg/s$

Only P and $\dot{P}$ are measured. The rest comes from assuming a spinning magnetic dipole — the straight lines in the diagram.

The *magnetars*

	ORDINARY PULSARS	SGRS / AXPS
P	10^{-3} – 10 s	2 – 12 s
$\dot{P}$	~10^{-15} s/s	10^{-13} – 10^{-10} s/s
L_X	~10^{30}–10^{33} erg/s	10^{32} – 10^{36} erg/s
t_c	> 10^{6} years	~10^{3} – 10^{4} years

SGR: *Soft Gamma Repeaters*. **AXP**: *Anomalous X-ray Pulsars*. Two classes that turned out to be one: in the ATNF catalogue, **25** magnetars have a measured period.

- **Slow**: $P \approx 2$–12 s, for a canonical neutron star (1.4 $M_\odot$, 10 km, 10^{45} g cm^2).
- **Too bright** in X-rays: $L_X > \dot{E}_{rot}$. Rotation cannot pay the bill.
- Who pays, in the standard model: **giant magnetic fields**, $B > 10^{14}$ G — above the quantum critical field, 4.4×10^{13} G.

27 DECEMBER 2004 · SGR 1806–20

A giant flare released ~10^{46} erg in 0.2 s — more than the Sun emits in 100,000 years. At its peak, ~7×10^{46} erg/s: **some 20 trillion suns**. It disturbed the Earth's ionosphere from ~28,000 light-years away.

A magnetar, *spinning*

SGR J1745−2900



field lines of a tilted dipole · real-time rotation

illustrative bursts

P 3.76 s B ≈ 2.6×10¹⁴ G characteristic age ≈ 3,400 yr $\dot{E}_{rot}$ ≈ 1.3×10³⁴ erg/s

bursts seen 1

EVENT	ENERGY
short bursts (~0.1 s)	10^{39} – 10^{41} erg
outbursts (months)	10^{41} – 10^{43} erg
giant flares	10^{44} – 10^{46} erg

The field lines, anchored in the crust, get twisted over time. When the crust can no longer take the stress, it cracks — a **starquake** — and the magnetic energy comes out all at once.

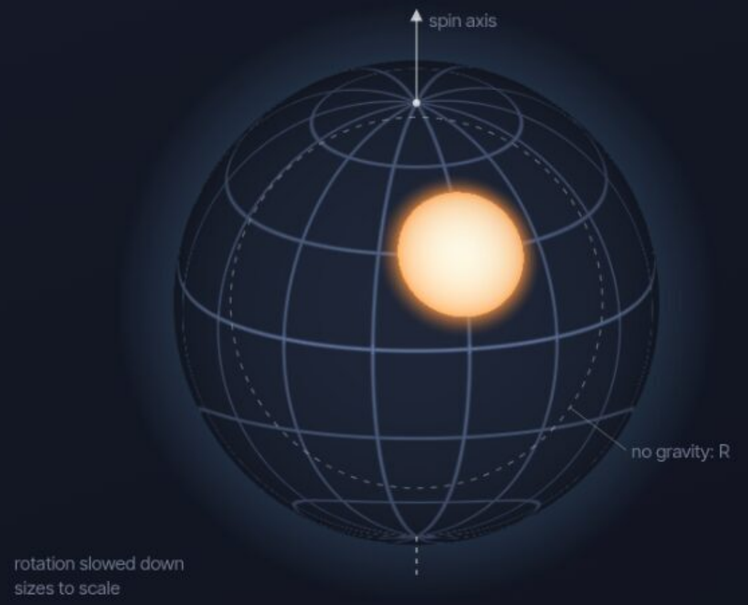
Giant flares in the Galaxy and the Magellanic Clouds: 1979, 1998 and 2004.

A HOT SPOT ON THE SURFACE

Light bends: *85% of the surface in view*

1.4 M⊙, 10 KM NEUTRON STAR · R/R_S = 2.4

no gravity | General Relativity





spot axis ↔ rotation, *i* 60°

line of sight ↔ rotation, *j* 75°

angular radius of the spot 20°

PF (GR) 0.93 PF (no gravity) 1.00 surface in view 85% limb ψ = 134.8° (α = 90°) spot always in view

THE HIGHEST DENSITY IN THE UNIVERSE

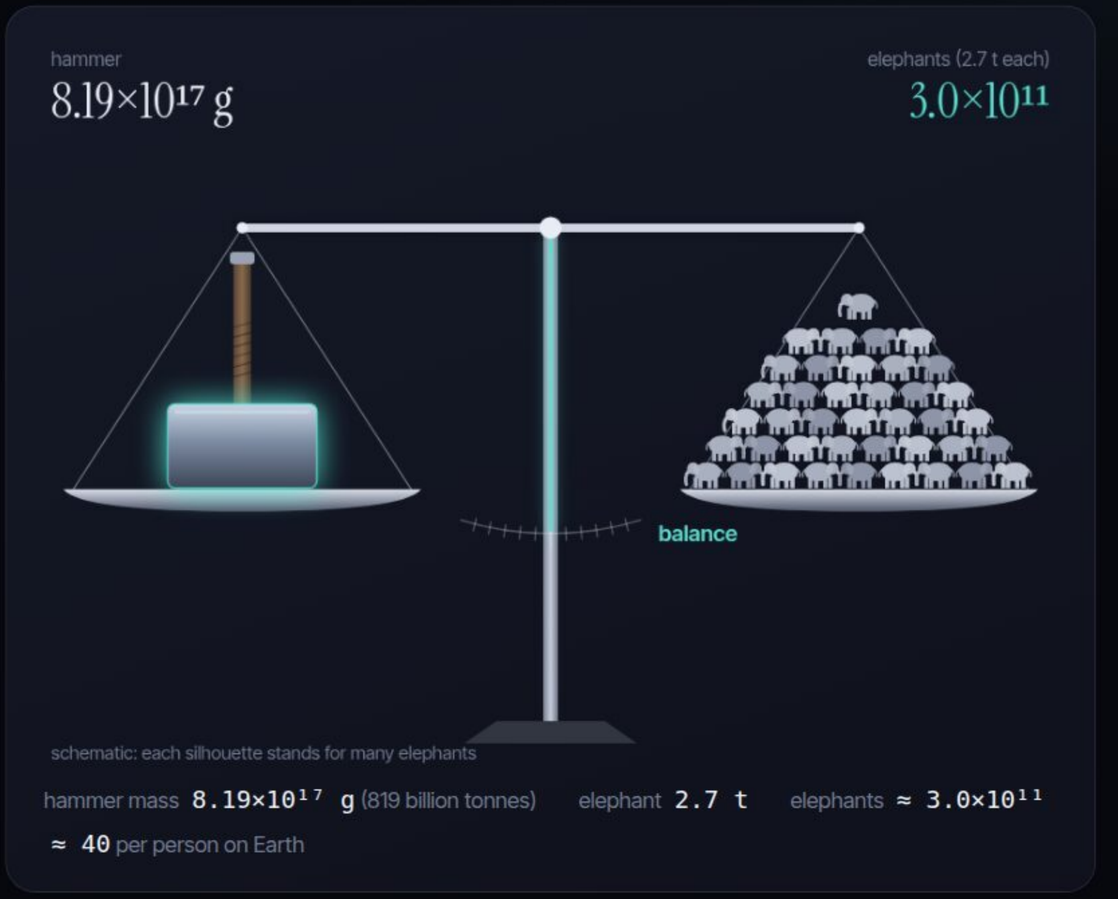
Forged in the core of *a neutron star*

> *"Thor Odinson, my heir, my firstborn. So long entrusted with the mighty hammer Mjölnir, forged in the heart of a dying star."*
>
> Odin, in *Thor* (Marvel Studios, 2011)

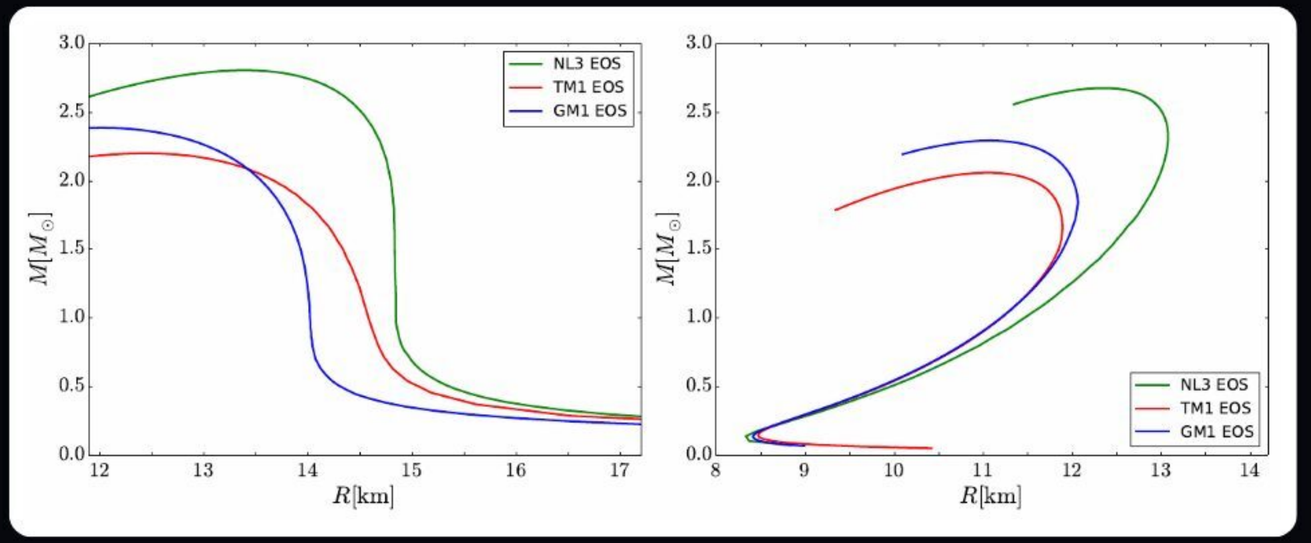
And what if it were forged in the core of a **neutron star**, at $\rho = 10^{14}\ \mathrm{g/cm^3}$? A head of $17.5 \times 26 \times 18$ cm:

$$M = \rho V = 10^{14}\ \frac{\mathrm{g}}{\mathrm{cm^3}} \times 8{,}190\ \mathrm{cm^3} = 8.19 \times 10^{17}\ \mathrm{g}$$

$$N_{\mathrm{elephants}} = \frac{8.19 \times 10^{17}\ \mathrm{g}}{2.7 \times 10^{6}\ \mathrm{g}} \approx \mathbf{300\ billion}$$



Mass and radius *with real nuclear physics*



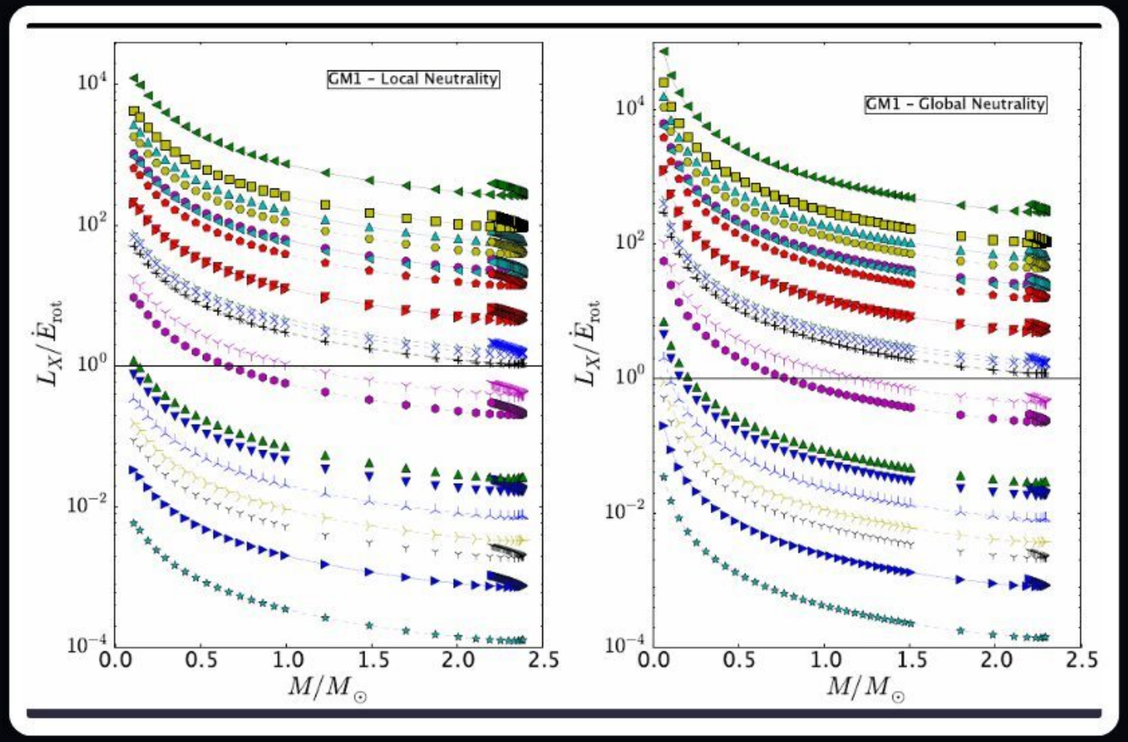
Mass–radius relation for the **NL3**, **TM1** and **GM1** equations of state, with local (left) and global (right) charge neutrality.

A neutron star is not a ball of free neutrons: the pressure comes from the **nuclear force**, described by relativistic mean-field models.

- Each equation of state gives a curve M(R) — and a maximum mass, between ~2 and 2.8 M☉.
- With these curves, M, R and the moment of inertia I are no longer canonical values: they are computed.

Figures in this section: R. C. R. de Lima and collaborators. Coelho, Cáceres, de Lima, Malheiro, Rueda & Ruffini, *A&A* 599, A87 (2017).

Magnetars *powered by rotation?*



$L_X/\dot{E}_{rot}$ against mass, for the known SGRs and AXPs, with the GM1 equation of state (local and global neutrality).

With realistic I(M) and R(M), $\dot{E}_{rot}$ changes. If $L_X/\dot{E}_{rot} < 1$, rotation is enough: **no giant field is needed**.

CONCLUSION: 9 (+3?) ROTATION-POWERED

$L_X/\dot{E}_{rot} < 1$: Swift J1834.9–0846, PSR J1846–0258, 1E 1547.0–5408, **SGR J1745–2900**, XTE J1810–197, PSR J1622–4950, SGR 1627–41, SGR 0501+4516, CXOU J171405.7–381031.

$L_X/\dot{E}_{rot} \sim 1$: SGR 1900+14, SGR 0418+5729, Swift J1822.3–1606.

DISCOVERED ON 25 APRIL 2013 · 0.1 PC FROM SGR A*

SGR J1745−2900, *the Galactic Centre magnetar*

START	PULSED FRACTION	kT_{BB} (KEV)	R_{BB} (KM)	L (10^{35} ERG/S)
12 May 2013	~0.38	0.88 ± 0.01	2.52 ± 0.09	4.9 ± 0.5
8–14 Oct 2016	~0.55	0.64 ± 0.03	0.78 ± 0.11	0.13 ± 0.02

Chandra, 0.3–8 keV. Coti Zelati et al., *MNRAS* 471, 1819 (2017): three and a half years of the outburst's decay.

$$\mathrm{PF} = \frac{F_{\max} - F_{\min}}{F_{\max} + F_{\min}}$$

3.76 s

period

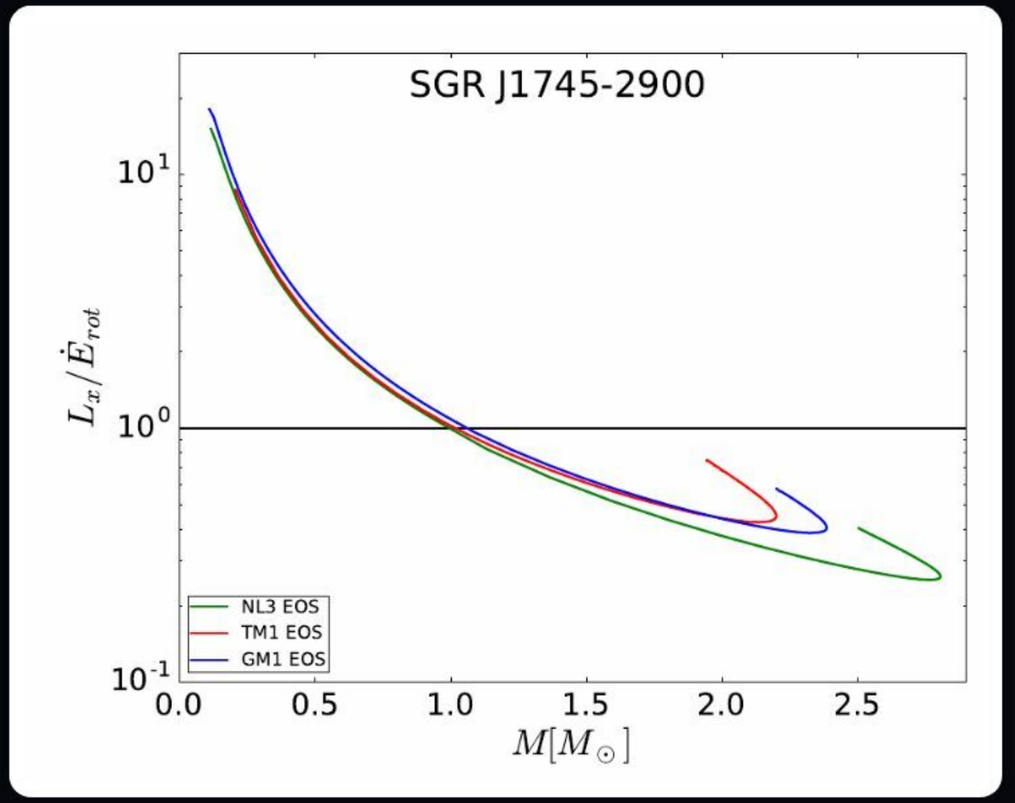
~5×10^{35} erg/s

in the discovery outburst (Swift)

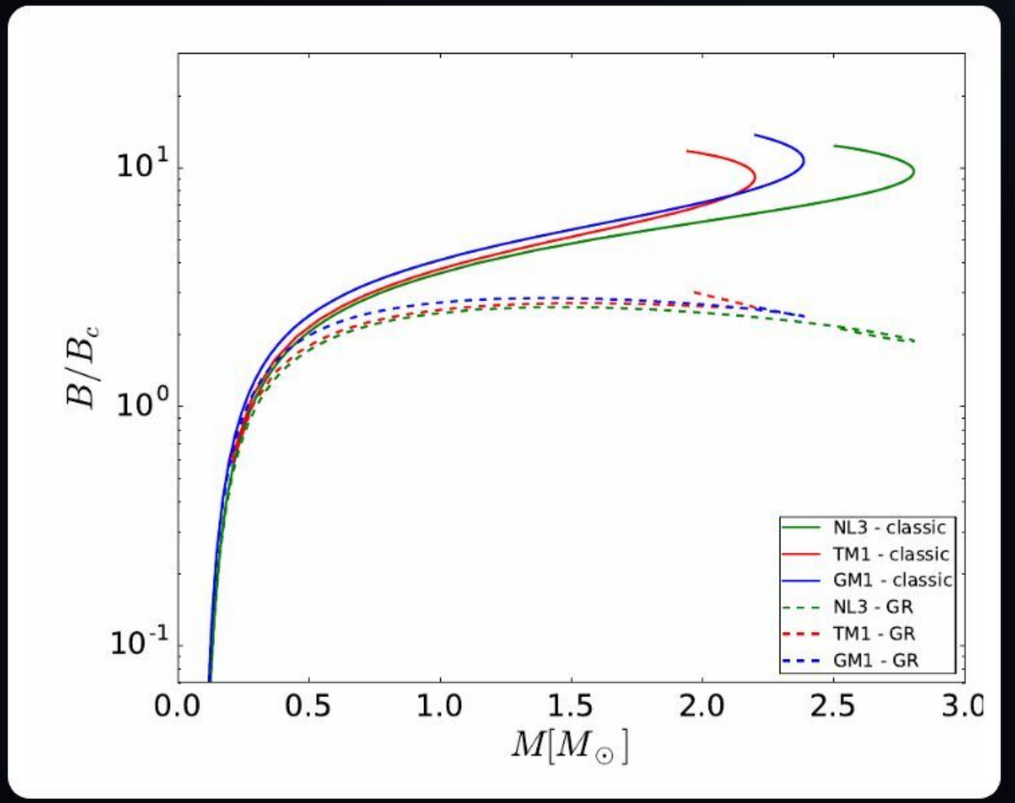
The hot spot **shrank** (from 2.5 to 0.8 km) and **cooled**, and the pulsed fraction **rose**. Can a real neutron star explain this?

Published version of this analysis: de Lima, Coelho, Pereira, Rodrigues & Rueda, *ApJ* 889, 165 (2020).

Two *checks*



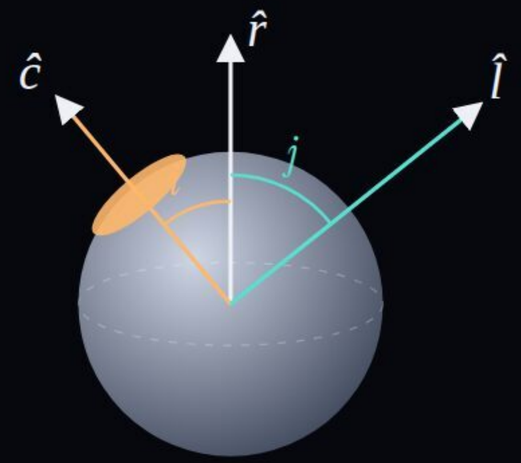
1st · efficiency. $L_X/\dot{E}_{rot} < 1$ for $M > 1.0\ M\odot$, in all three equations of state: **rotation-powered**.



2nd · magnetic field at the surface, in units of $B_c = 4.41\times10^{13}$ G: classical calculation (solid) and with General Relativity (dashed).

TUROLLA & NOBILI 2013 · BELOBORODOV 2002

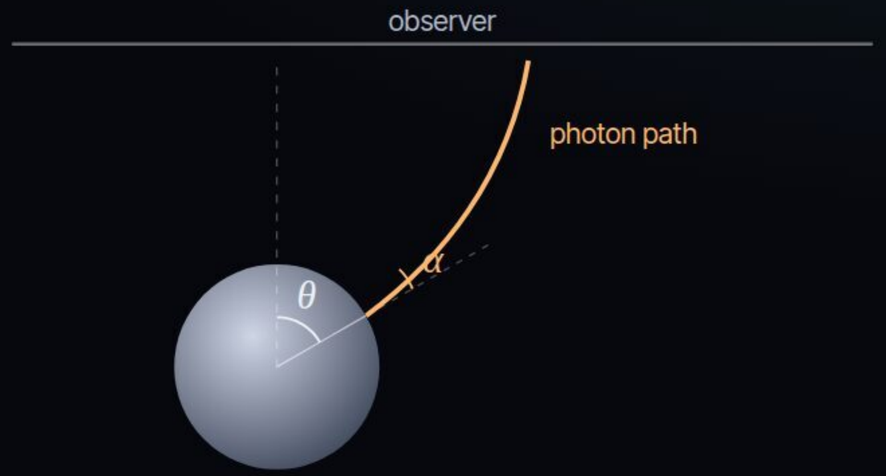
The pulsed-flux *model*



r̂ rotation axis · ĉ axis of the emitting cap · Î line of sight

$$i = \arccos(\hat{r}\cdot\hat{c}) \quad \cdot \quad j = \arccos(\hat{r}\cdot\hat{I})$$

- Circular spots of any size, anywhere on the surface (Turolla & Nobili 2013).
- M and R come from the equations of state (GM1, TM1, NL3).
- Schwarzschild exterior: the rotation is slow (P ≈ 2–10 s) and its effects on the metric are neglected.



$$1 - \cos\alpha = (1 - \cos\theta)\,(1 - R_s/R)$$

BELOBORODOV'S APPROXIMATION (2002)

$$F_\nu = (1 - R_s/R)^2\, B_\nu(T)\, A_{\text{eff}}, \quad A_{\text{eff}} = R^2\,[\,\frac{R_s}{R}\, I_s + (1 - \frac{R_s}{R})\, I_p\,]$$

FLUX FROM A BLACK-BODY SPOT · EFFECTIVE AREA

What the code *computes*

MAIN OUTPUT

- map of pulsed fractions in the (i, j) plane;
- light curve: normalised flux against rotational phase.

SIDE CALCULATIONS

- angular opening of the polar cap (light cylinder);
- Beloborodov angle and visible area of the star;
- maximum possible pulsed fraction.

VALIDITY

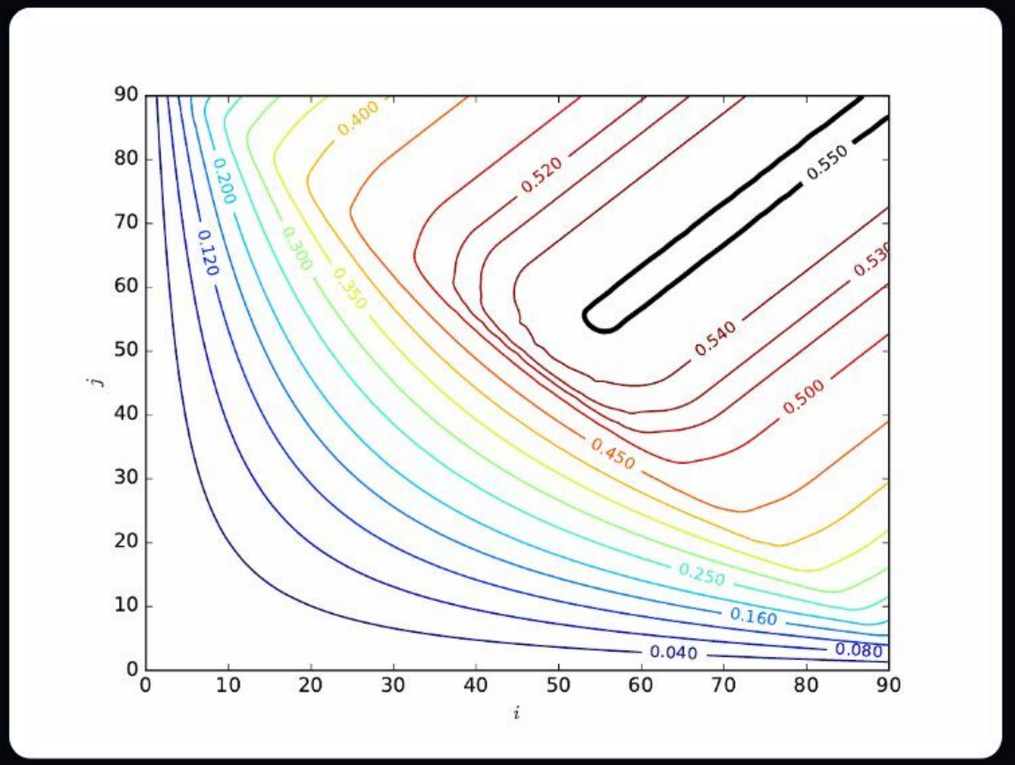
Error < 3% for $R > 3R_s$ (here, $R/R_s = 3.38$), with $R_s = 2GM/c^2$.

VARIABLE	DEFINITION	SOURCE
M, R	mass and radius	equation of state
N_s, θ_c, T	number, half-opening and temperature of the spots	black-body fit
θ_d	angular distance between the spots	pulsed fraction
i, j	viewing angles	PF and pulse profile

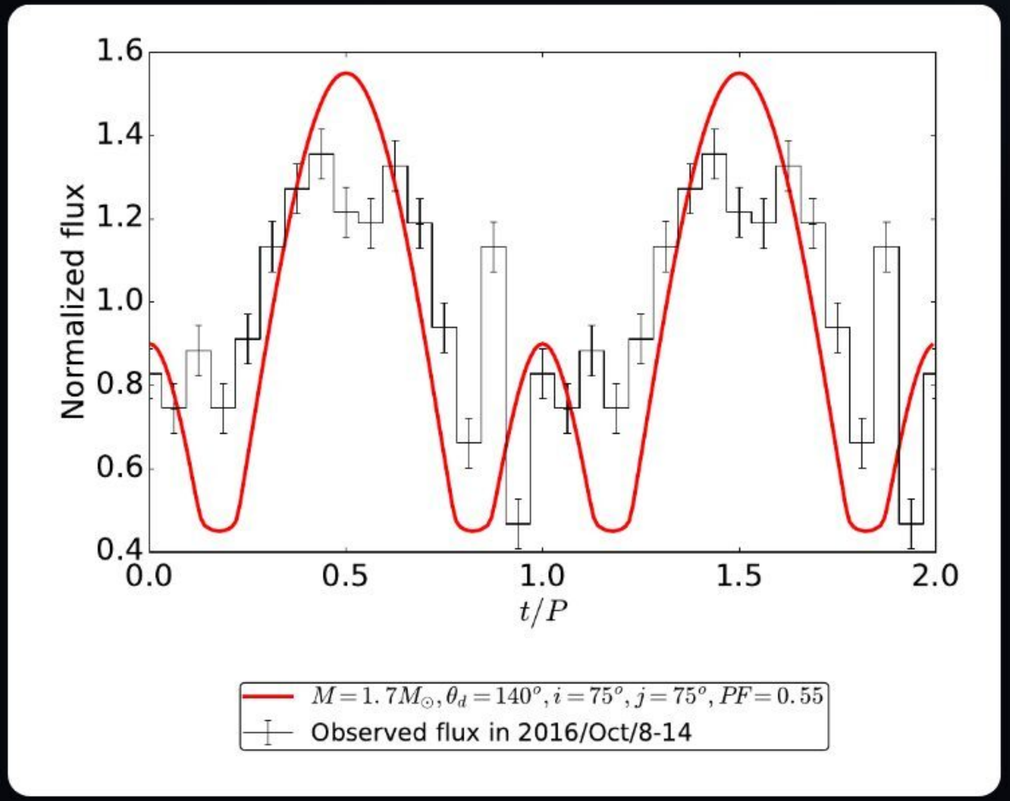
BELOBORODOV CLASSES (2002)

I · the antipodal spot never appears, and the primary is seen all the time. **II** · the primary always, and the antipodal for part of the time. **III** · the primary disappears for a fraction of the period. **IV** · both are seen all the time.

Pulsed fraction *and light profile*



PF map in the (i, j) plane. The thick black curve is PF = 0.55, the value observed in 2016.



Pulse profile: the model (red) with M = 1.7 M☉, θ_d = 140°, i = j = 75°, against the flux observed by Chandra on 8–14 October 2016.

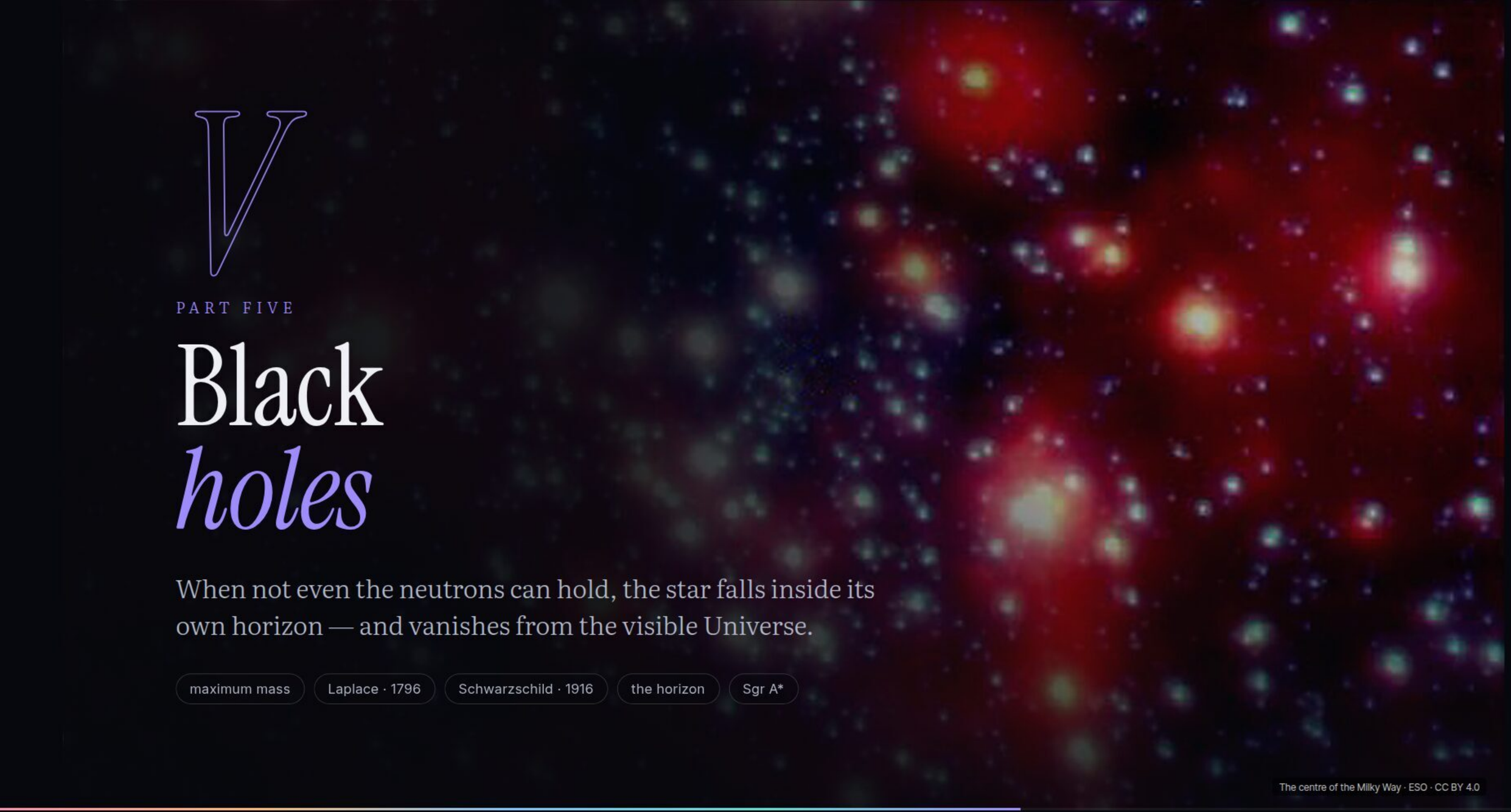
V

PART FIVE

Black *holes*

When not even the neutrons can hold, the star falls inside its own horizon — and vanishes from the visible Universe.

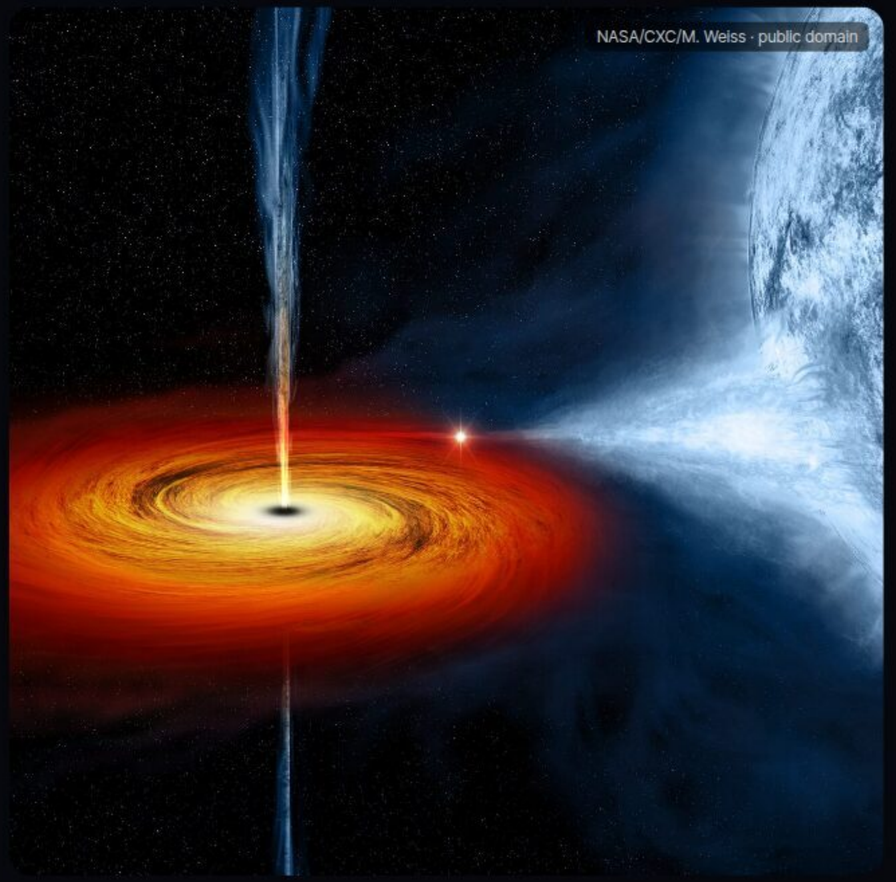
maximum mass · Laplace · 1796 · Schwarzschild · 1916 · the horizon · Sgr A*



The centre of the Milky Way · ESO · CC BY 4.0

TOLMAN · OPPENHEIMER · VOLKOFF · 1939

$M_{max} \approx 2\,M\odot$ — *and beyond?*



Neutron stars also have a maximum mass. Above ~2–2.3 M☉, neither neutron pressure nor the nuclear force can hold the star up.

2.08 M☉

PSR J0740+6620, the heaviest measured precisely

~21 M☉

Cygnus X-1, the first accepted black hole (1972)

But what if the neutron star gains more mass — *accreting* gas from a companion — and goes over M_{max}?

Nothing can *stop the collapse*

No known force can hold the star up.

And what if there is some physics we don't know yet, strong enough to halt the collapse?

It makes no difference: once the *event horizon* forms, it is too late. Inside, every future points to the centre.

1783 – 2019

A brief history *of black holes*

1783

Michell: “dark stars”, from which light cannot escape

1796

Laplace arrives at the same idea

1916

Schwarzschild: the first exact solution of GR

1935

Eddington: “there should be a law of Nature…”

1939

Oppenheimer & Snyder: the collapse, calculated

1963

Kerr: black holes that spin

1967

Wheeler popularises the name “black hole”

2019

EHT: the first image, of M87*

A *dark* star



Pierre-Simon Laplace
1749 – 1827

By Newton's physics, light could not escape from a sufficiently massive and compact object.

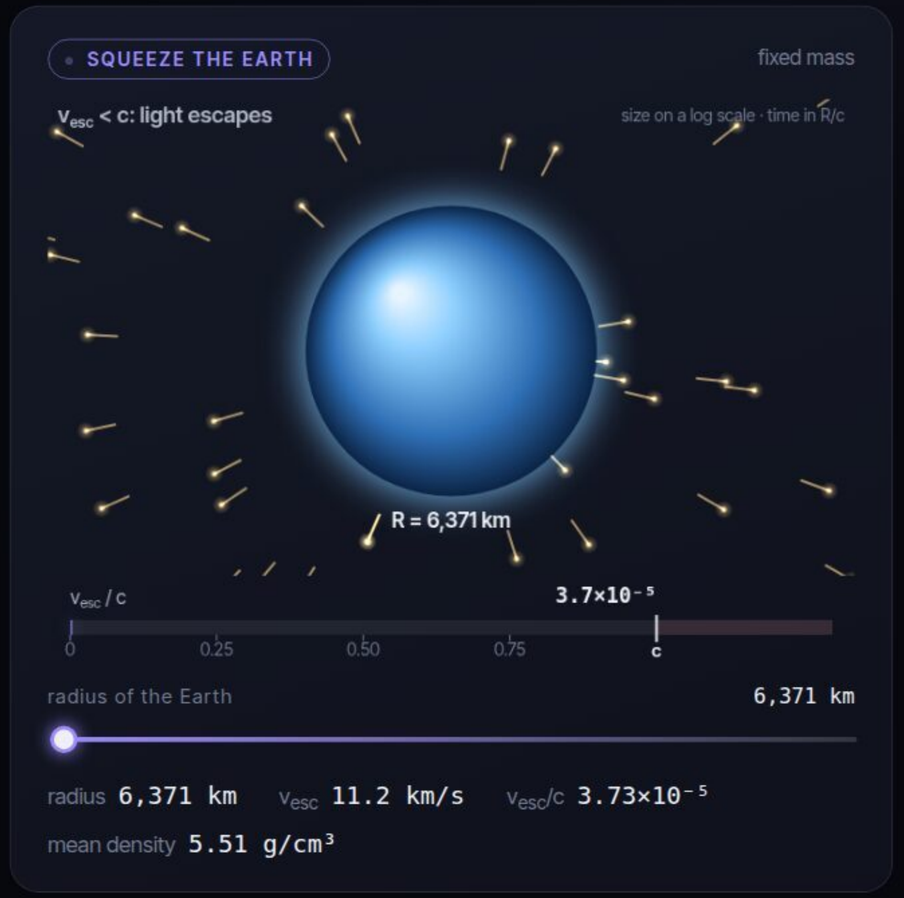
$$\frac{mv^2}{2} - \frac{GMm}{R} = 0 \Rightarrow v_{esc} = \frac{2GM}{R}^{1/2}$$

INITIAL ENERGY = FINAL ENERGY

On Earth, v_{esc} = 11.18 km/s. At what r would the escape velocity equal the speed of light?

$$r = \frac{2GM}{c^2}$$

Earth: **8.9 mm**. Sun: **2.95 km**.

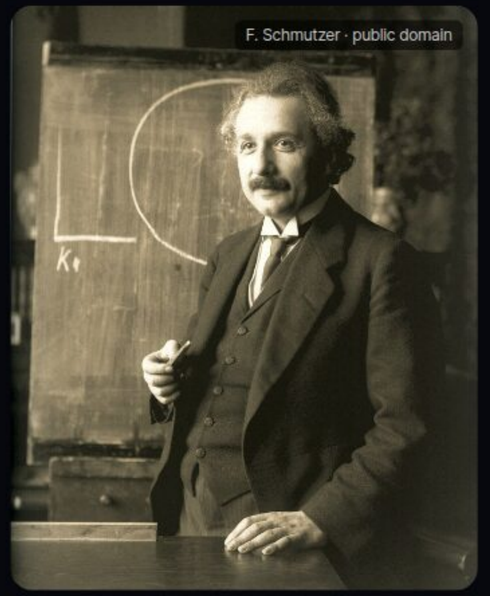


DECEMBER 1915 · THE RUSSIAN FRONT

A month after *Einstein*



Karl Schwarzschild
1873 – 1916



Albert Einstein
1921

Barely a month after Einstein presented General Relativity (25 November 1915), Schwarzschild — serving in the artillery on the Russian front — found an **exact analytical solution** of the equations.

> *"I have read your paper with the utmost interest. I had not expected that one could formulate the exact solution of the problem in such a simple way. I liked very much your mathematical treatment of the subject."*
>
> Einstein to Schwarzschild, 9 January 1916

Schwarzschild died in May 1916, of an illness he contracted at the front.

SPHERICAL, UNCHARGED, NON-ROTATING

The first exact solution *was already a black hole*

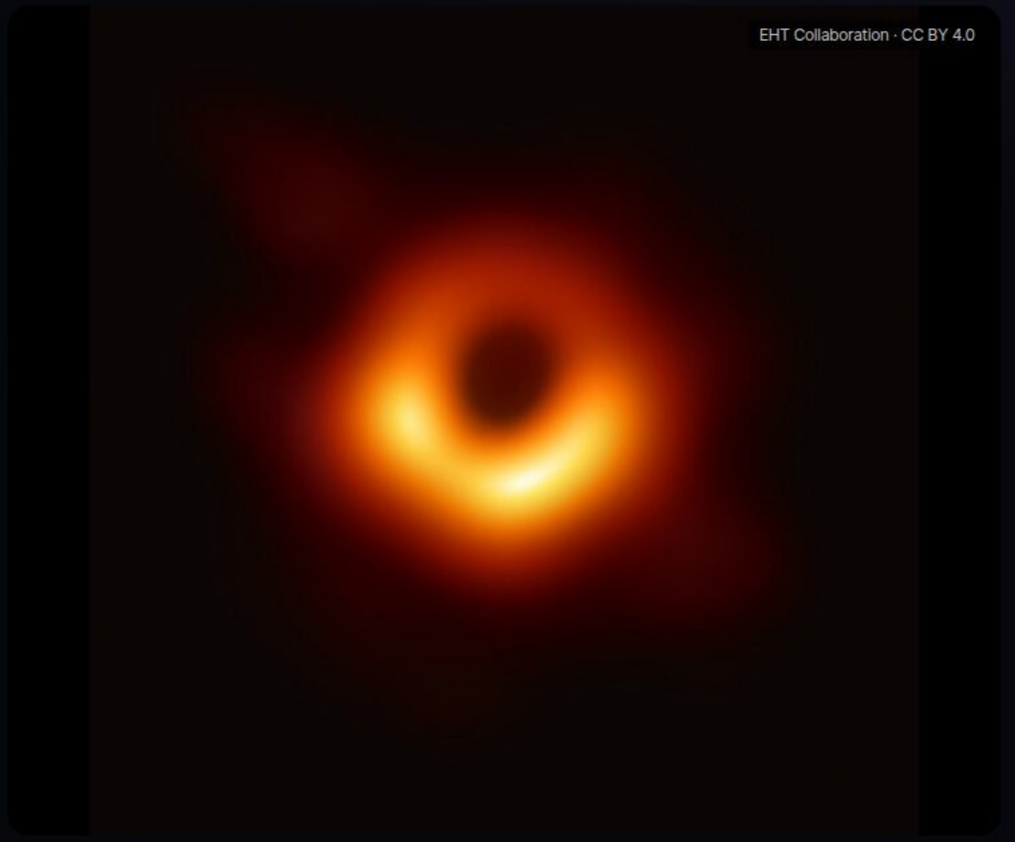
WHAT NEITHER OF THEM KNEW

The solution was **complete**: besides the exterior of a star, it described a spherical, neutral, non-rotating black hole — the **Schwarzschild black hole**.

UNTIL THEN

Einstein had only looked for approximate solutions — the post-Newtonian formalism — and used them to explain the **bending of light by the Sun** and the **precession of Mercury**.

It took some 50 years for the community to accept that such a thing really existed.



The shadow of M87*, Event Horizon Telescope, 2019 — 103 years later.

ROYAL ASTRONOMICAL SOCIETY · 11 JANUARY 1935

Eddington versus *Chandrasekhar*



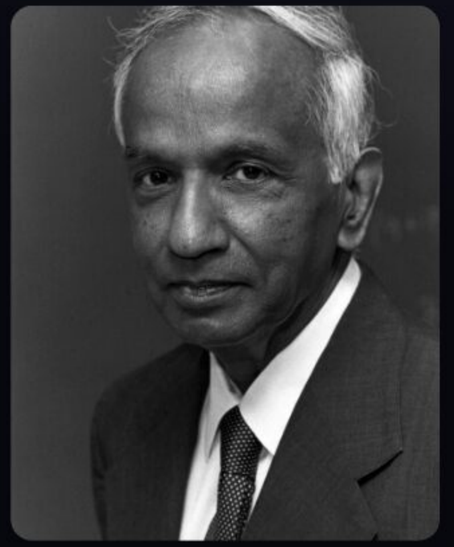
Arthur Eddington
1882 – 1944

One of the first to understand General Relativity, Eddington saw it at once: if Chandrasekhar was right about the mass limit, massive stars would **inevitably** form black holes.

> *"I think there should be a law of Nature to prevent a star from behaving in this absurd way!"*
>
> Arthur Eddington, Royal Astronomical Society, 11 January 1935

Chandrasekhar would later regret that it took **almost thirty years** for the significance of his result to be recognised.

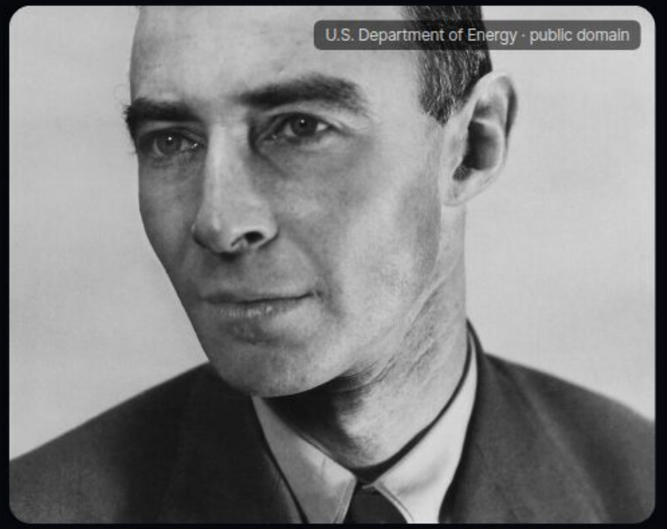
K. C. Wali, *Chandra: a biography of S. Chandrasekhar* (1991).



Chandrasekhar
aged 24 in 1935

1939 · 1963 · 1967

The collapse, the spin *and the name*



1939 · Oppenheimer

With Snyder, the first rigorous calculation of a star collapsing all the way to a black hole. With Volkoff, the maximum mass of neutron stars.



1963 · Roy Kerr

Another exact solution of Einstein's equations: a spinning black hole. It is what nature should produce.



1967 · John A. Wheeler

Popularises the term "black hole" — to the dismay of the French, for whom *trou noir* sounded indecent.

THE SCHWARZSCHILD METRIC

Three slices *of the same formula*

$$\Delta s^2 = -\left(1 - \frac{2GM}{c^2 r}\right)(c\Delta t)^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1}(\Delta r)^2 + r^2\left[(\Delta\theta)^2 + \sin^2\theta\,(\Delta\varphi)^2\right]$$

M = MASS OF THE CENTRAL OBJECT

$\Delta T = \Delta\Theta = \Delta\Phi = 0$

$$\Delta r_p = (1 - 2GM/c^2 r)^{-1/2}\,\Delta r$$

Radial distances measured with a ruler: larger than Δr.

$\Delta R = \Delta\Theta = \Delta\Phi = 0$

$$\Delta\tau = (1 - 2GM/c^2 r)^{1/2}\,\Delta t$$

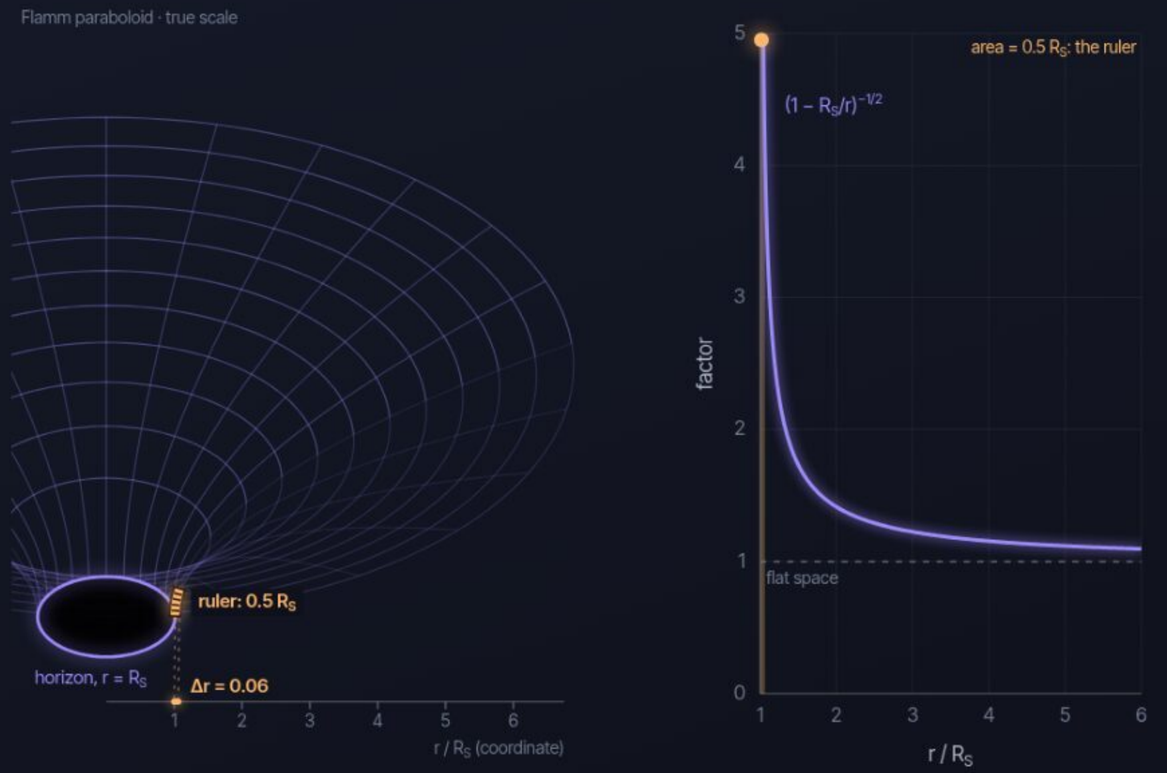
The time on a clock at rest: it runs slower than one far away.

$\Delta T = \Delta R = 0$

$$\Delta S = r\,[(\Delta\theta)^2 + \sin^2\theta\,(\Delta\varphi)^2]^{1/2}$$

The angular part is the familiar one: the distance between two points on the Earth's surface ($\theta = 90° - \text{latitude}$).

The ruler *in the well*



ruler at r `1.02 Rs` factor `8.08` Δr covered `0.061 Rs` to the horizon `0.25 Rs`

- r is the coordinate of an observer at infinity: the circumference divided by 2π.
- Δr_p is the distance a local observer measures with a ruler.
- Distances are **larger** than they would be in flat space: between the spheres of circumference $2\pi r_1$ and $2\pi r_2$ there is more than $r_2 - r_1$.
- The factor $(1 - r_s/r)^{-1/2}$ diverges at r_s, but the distance to the horizon is **finite**: from $3r_s$ down to r_s it is $3.6\, r_s$, not 2.

On the left, Flamm's paraboloid: the equatorial slice (constant t) drawn as a surface with the same geometry.

THE EVENT HORIZON

The *Schwarzschild* radius

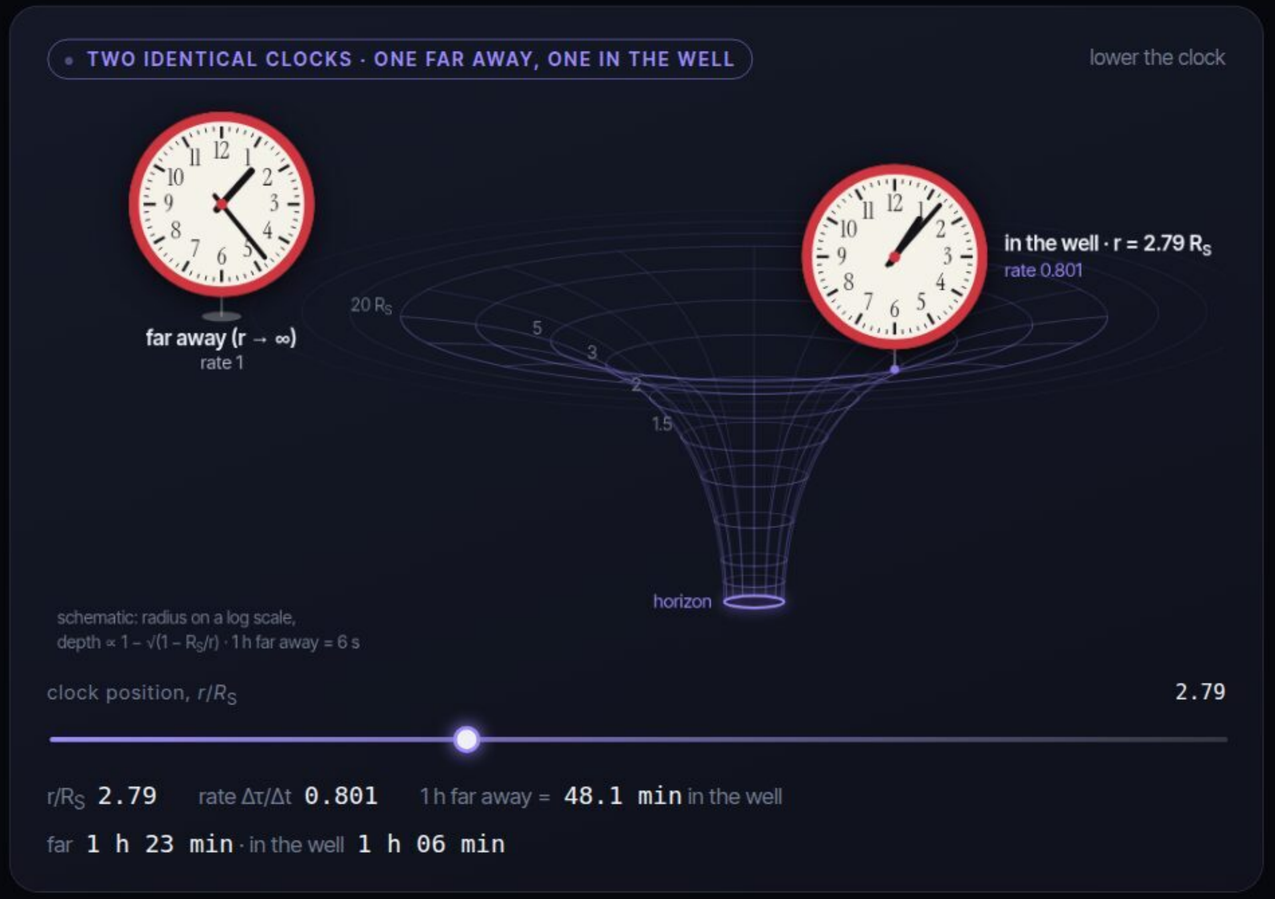
$$R_S = \frac{2GM}{c^2}$$

2.95 KM PER SOLAR MASS

Where the factor $(1 - 2GM/c^2r)$ in the metric vanishes. There is nothing there — no surface, no wall: it is a **causal** boundary. What goes in does not come out.

OBJECT	MASS	R_S
Earth	6×10^{24} kg	8.9 mm
Sun	1 $M_\odot$	2.95 km
neutron star (radius 12 km)	1.4 $M_\odot$	4.1 km
Cygnus X-1	21 $M_\odot$	62 km
Sgr A*	4.3×10^{6} $M_\odot$	12.7 million km · 18 $R_\odot$
M87*	6.5×10^{9} $M_\odot$	1.9×10^{10} km · 128 AU

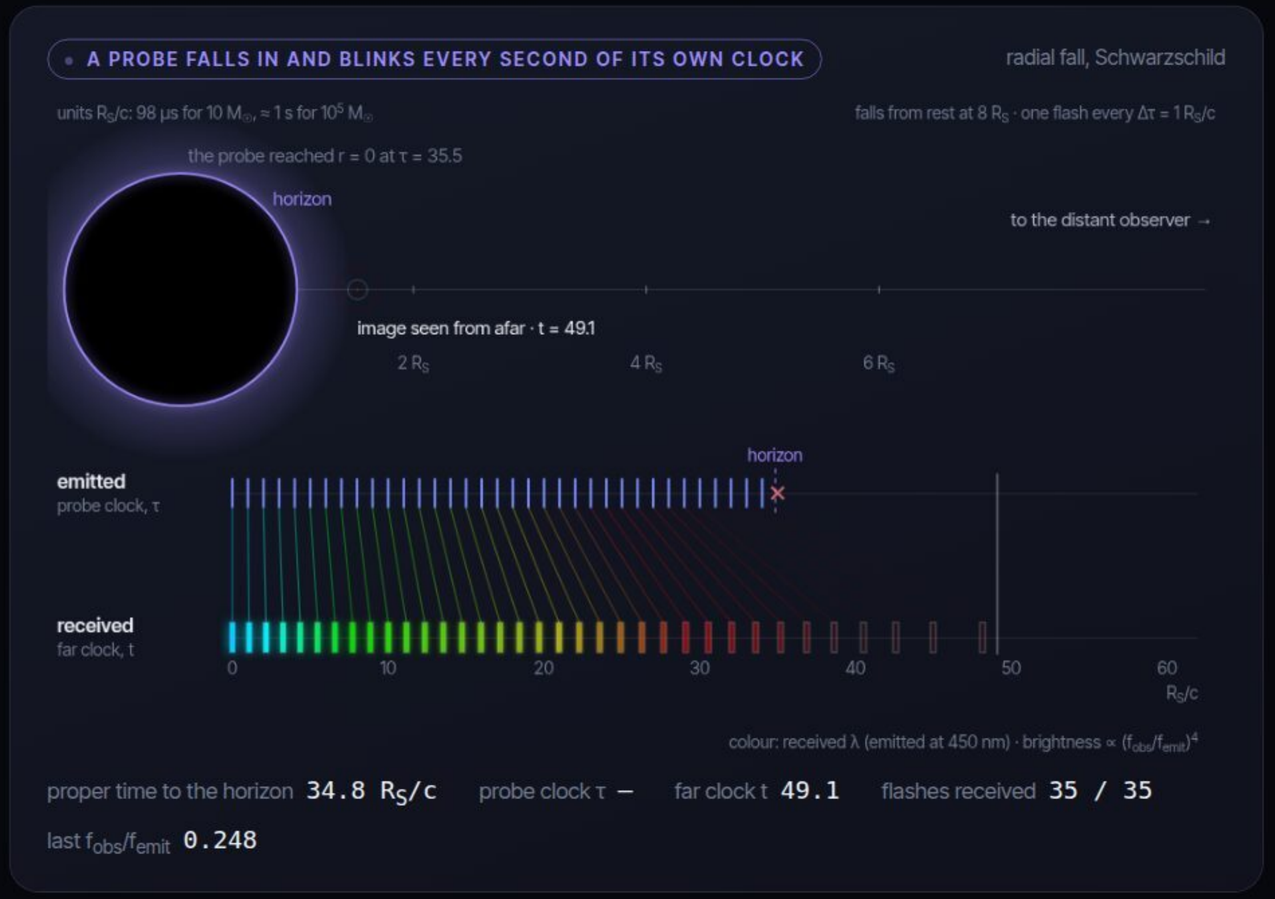
Time *in the well*



$$\Delta\tau = \left(1 - \frac{2GM}{c^2 r}\right)^{1/2} \Delta t$$

- The clock deep in the well runs slower, as seen from far away — and it is no illusion: when it comes back up, it shows less time.
- GPS: +45 μs/day from altitude, −7 μs/day from the satellites' speed. Without the correction, an error of ~10 km a day.
- At the surface of a neutron star: $\Delta\tau/\Delta t \approx 0.8$ — some 12 minutes lost every hour.
- At the horizon, $\Delta\tau/\Delta t \to 0$.

Seen from afar, *time freezes*



An observer at infinity would see time **freeze** at R_S. Does that mean light stops?

No! It means it is **trapped**. For photons $\Delta s^2 = 0$; if $\Delta\tau \to 0$, the spatial terms vanish too ($\Delta r = \Delta\theta = \Delta\phi = 0$): the photon does not leave that region.

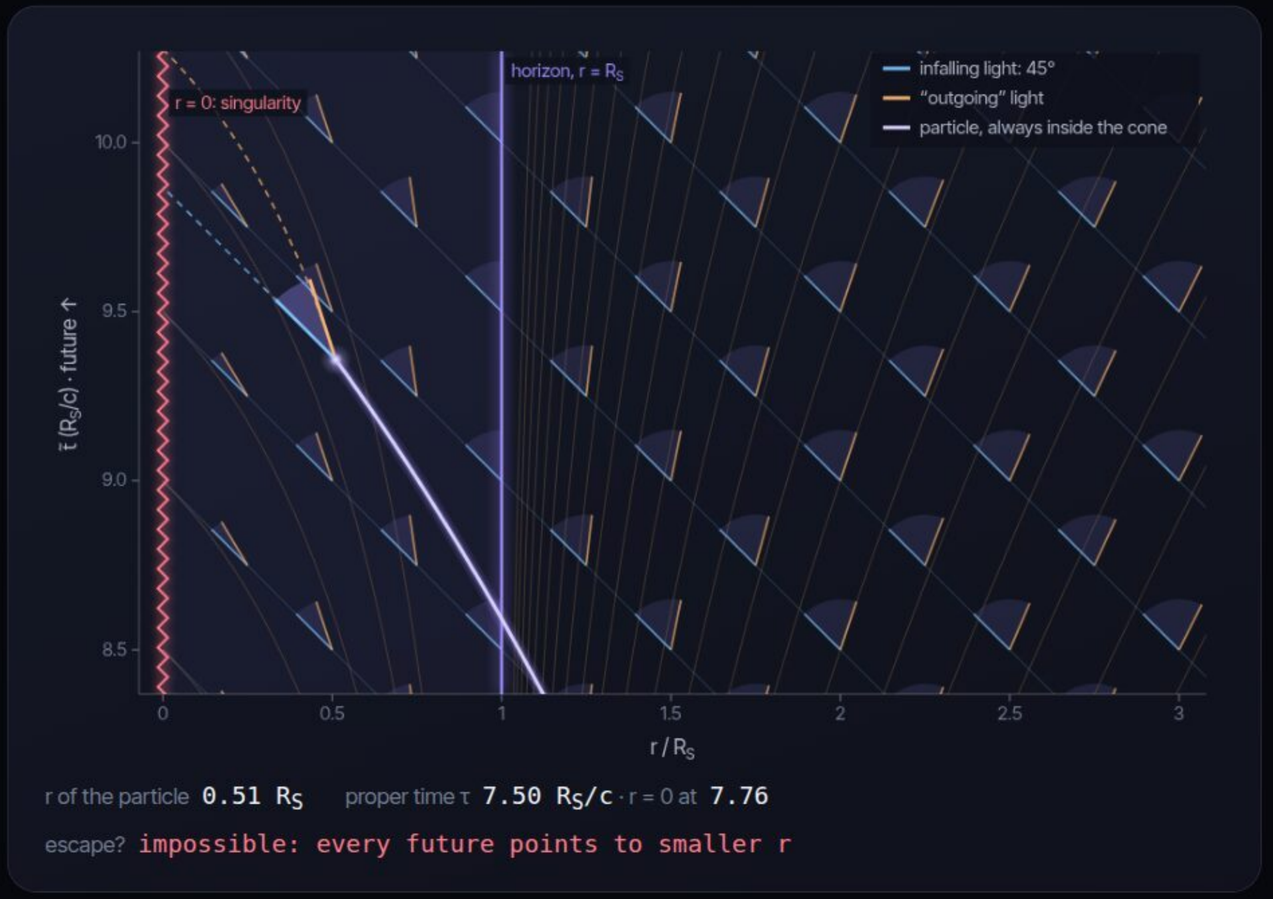
PHOTONS: OUR NATURAL CLOCKS

A wave of frequency f is a clock that ticks $1/\Delta t$ times per second:

$$f_\infty = (1 - 2GM/c^2 r)^{1/2}\, f_{\text{local}}$$

Whoever falls in notices nothing special when crossing the horizon.

Every future *points to the centre*



r of the particle `0.51 Rs` proper time τ `7.50 Rs/c` · r = 0 at `7.76`

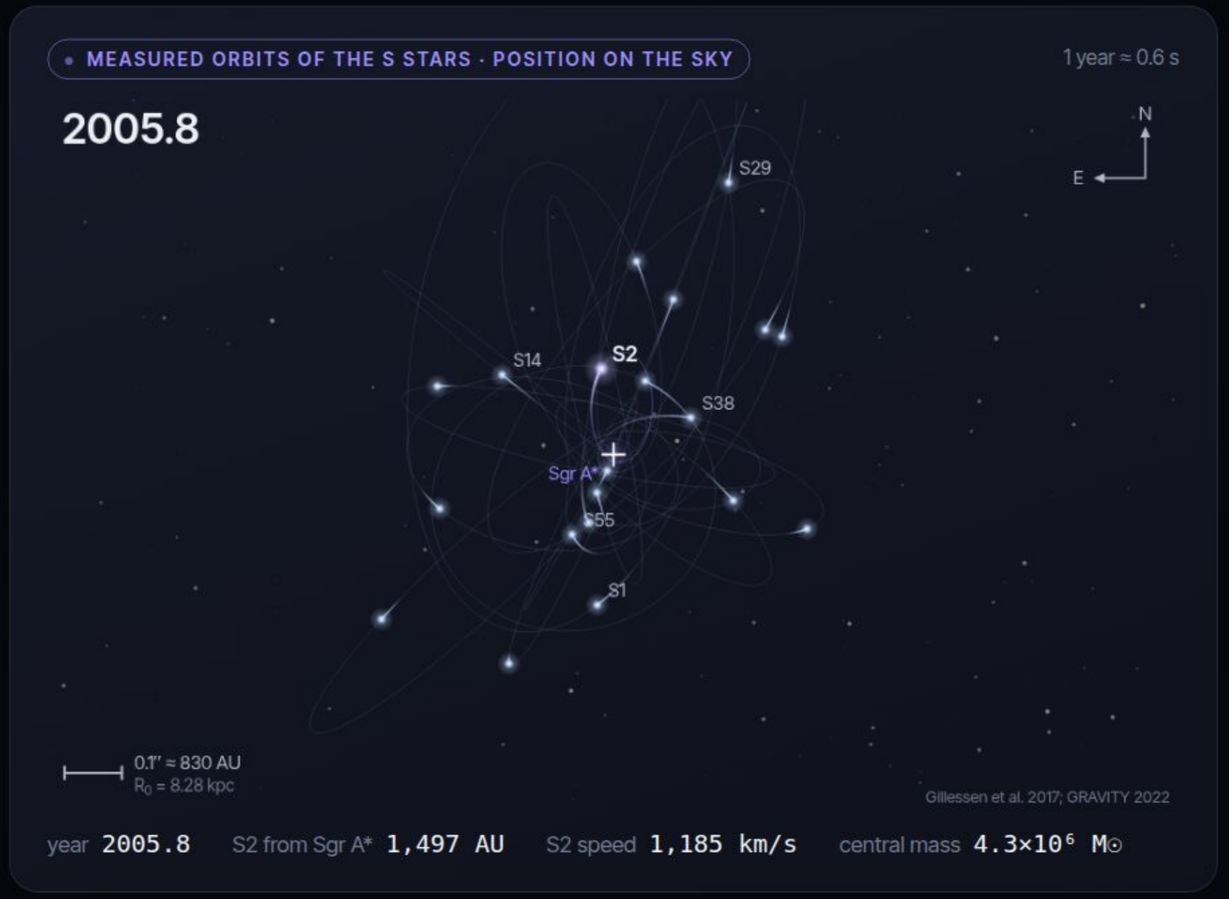
escape? `impossible: every future points to smaller r`

Matter must move **inside** the light cones.

- Far away, the cone is the familiar one: you can go outwards or inwards.
- Near the horizon, it tips inwards, but still leaves a way out.
- At the horizon, the outer edge becomes vertical: "outgoing" light stands still.
- Inside, both sides of the cone point to smaller r: **moving towards the future means moving towards the centre**.

Eddington–Finkelstein coordinates: $\tilde{t} = t + r_s \ln|r/r_s - 1|$; infalling light follows straight lines at 45°.

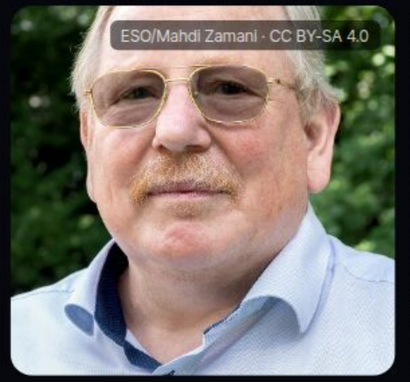
Sagittarius A*: *stars orbiting nothing*



4.3×10^6 $M_\odot$
mass at the centre

16 years
period of S2

S2 passes ~120 AU from the centre at ~7,700 km/s, 2.6% of c. In 2018 and 2020 the VLT measured its gravitational redshift and its Schwarzschild precession.



Reinhard Genzel
VLT · since 1992

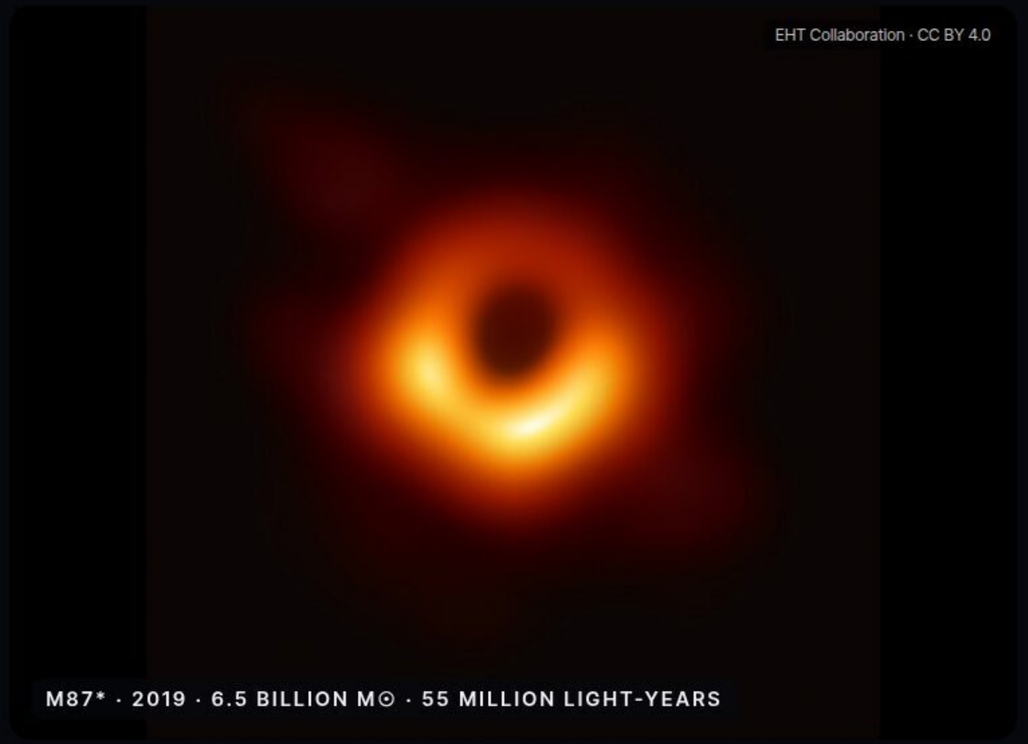


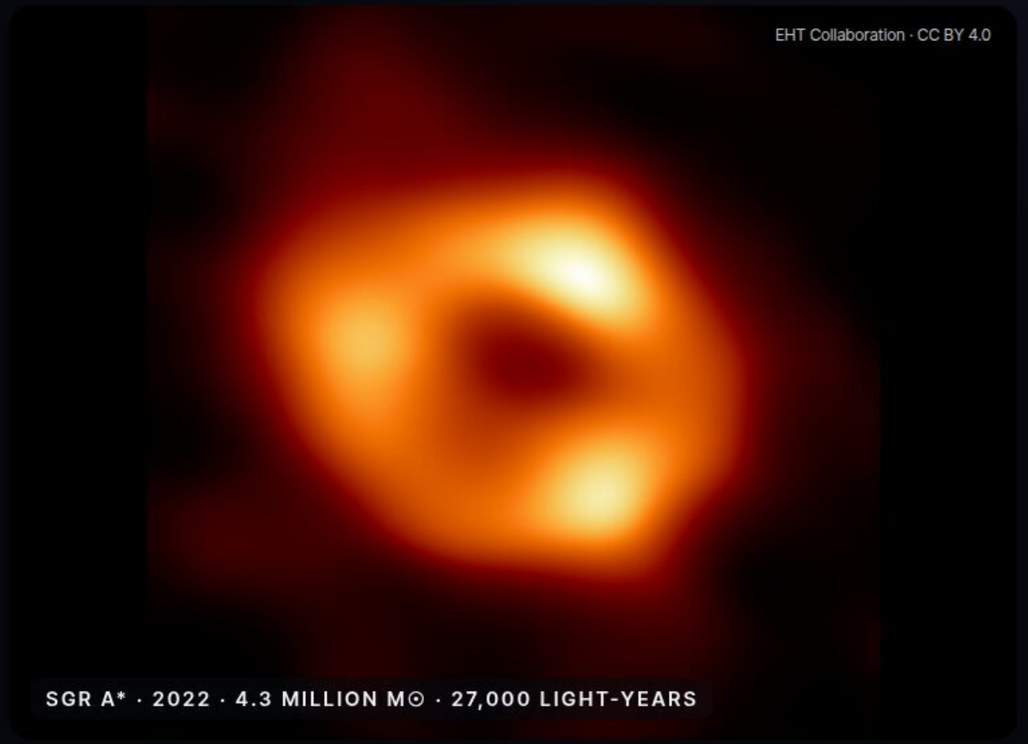
Andrea Ghez
Keck · since 1995

Nobel Prize 2020, with Roger Penrose. Orbits: Gillessen et al., *ApJ* 837, 30 (2017).

EVENT HORIZON TELESCOPE · 2019 AND 2022

The *shadows*

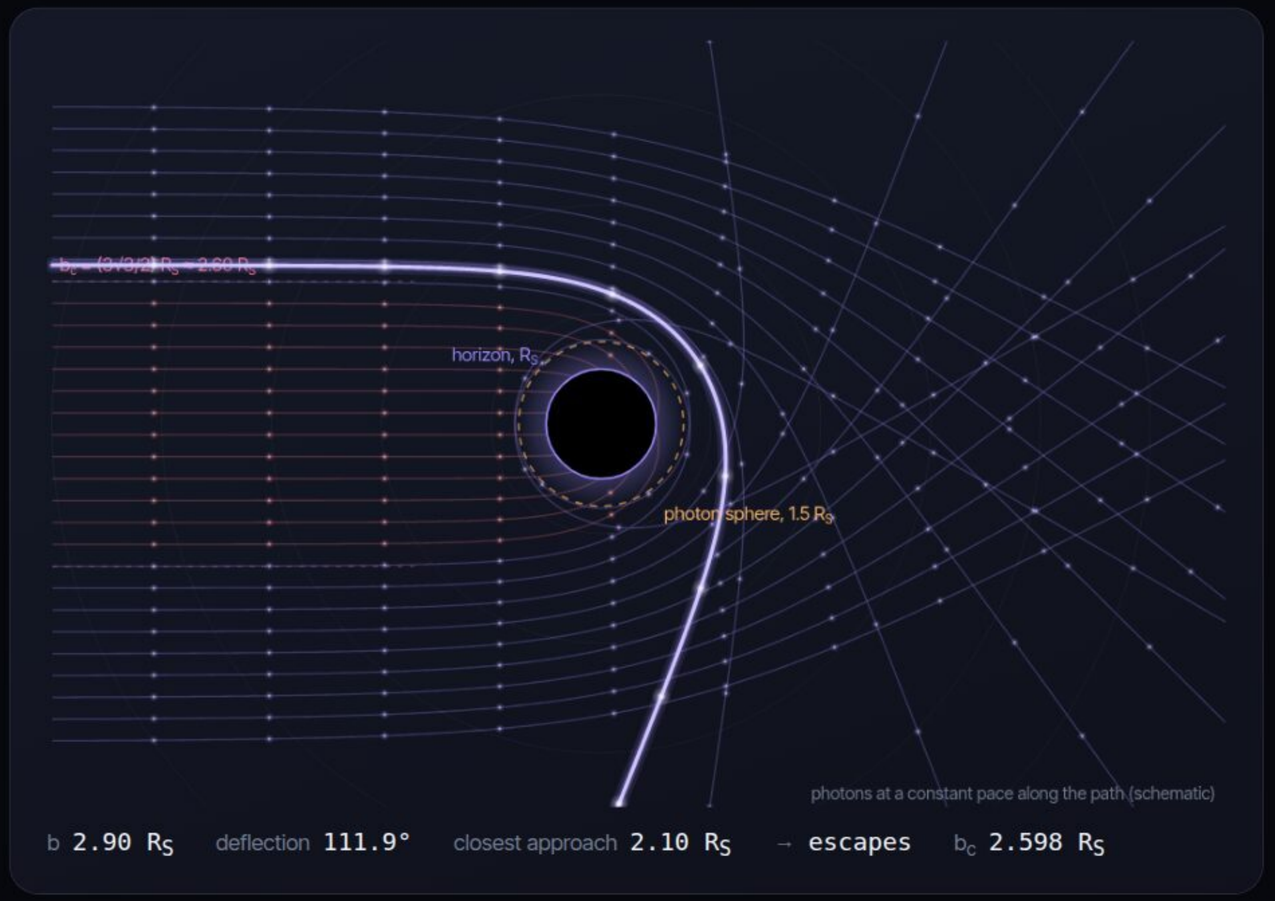




The ring is ~5 R_S across: it is the photon sphere, at 1.5 R_S, seen through the lens of the black hole itself — shadow diameter = $2\sqrt{27}\,GM/c^2 \approx 5.2\,R_S$.

SCHWARZSCHILD GEODESICS

How light *bends*



b **2.90** R_S deflection **111.9°** closest approach **2.10** R_S → **escapes** b_c **2.598** R_S

$$u'' + u = \frac{3}{2} R_S u^2$$

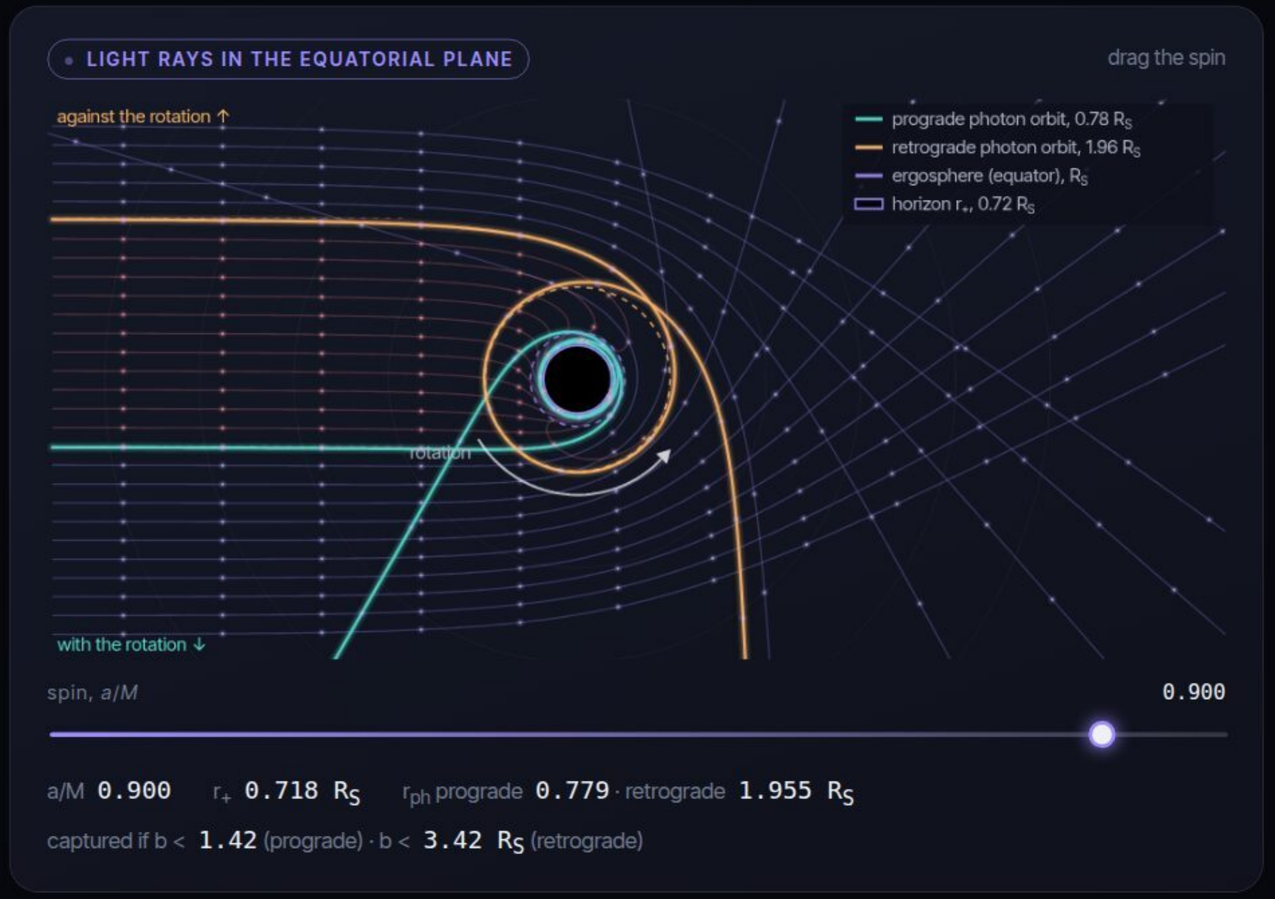
$U = 1/R$ · ORBIT EQUATION FOR LIGHT

- $b > 3\sqrt{3}\,GM/c^2 \approx 2.6\ R_S$: the light is deflected and escapes.
- $b = 2.6\ R_S$: unstable orbit on the **photon sphere**, $r = 1.5\ R_S$.
- $b < 2.6\ R_S$: captured.

The term on the right is the General Relativity correction. Without it, light would travel in a straight line.

KERR GEODESICS · 1963

The black hole that *spins*



- The rotation **drags** the spacetime around it (the Lense–Thirring effect).
- Light travelling with the rotation gets closer before being captured: the photon orbit goes from 1.5 R_S down to 0.5 R_S as $a \to M$.
- Against the rotation, the orbit moves out, up to 2 R_S.
- Capture becomes asymmetric: the shadow shifts and flattens on one side.

Kerr null geodesics in Boyer–Lindquist coordinates, integrated in Mino time.

R = 0

The *singularity*

$$\Delta s^2 = -\left(1 - \frac{2GM}{c^2 r}\right)(c\Delta t)^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1}(\Delta r)^2 + r^2[(\Delta\theta)^2 + \sin^2\theta\,(\Delta\varphi)^2] \quad r = 0 \rightarrow \text{singularity}$$

$R = R_S$ IS NOT SINGULAR

The metric seems to blow up there, but that is just a bad choice of coordinates — like longitude at the North Pole. Whoever falls in crosses the horizon without noticing, and the curvature there is finite.

$R = 0$ IS

The curvature diverges — the Kretschmann invariant goes as $48G^2M^2/c^4r^6$ — and tidal forces stretch anything apart. General Relativity **predicts its own limit**: a quantum theory of gravity has to take over there, and we do not have one yet.

RECAP

Three masses *decide the end*

M_J

Jeans

Above it, a cold cloud collapses, fragments and lights up stars. It shrinks as the cloud gets denser.

1.44 $M_\odot$

Chandrasekhar

Above it, degenerate electrons cannot hold: there is no white dwarf, and the iron core collapses.

~2.2 $M_\odot$

Tolman–Oppenheimer–Volkoff

Above it, not even neutrons can hold: a horizon forms, and the star becomes a black hole.

The whole life of a star is gravity on one side and, on the other, heat, fusion and — in the end — *quantum mechanics*.

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Research figures (Part IV) — R. C. R. de Lima and collaborators; Coelho et al., A&A 599, A87 (2017); de Lima et al., ApJ 889, 165 (2020)

P–Ṗ diagram — ATNF pulsar catalogue 2.8.1 (Manchester et al., AJ 129, 1993, 2005)

Orbits around Sgr A* — Gillessen et al., ApJ 837, 30 (2017); GRAVITY Collaboration, A&A 657, L12 (2022)

THANK YOU

Questions?

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Rafael C. R. de Lima

rafael.lima@udesc.br · rafael-lima.pages.dev