DARK MATTER IN NEUTRON STARS

Simulating the *invisible*



How a neutron star captures dark matter *one collision at a time* — and what state the captured particles end up in.

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$\mathscr{E} = \gamma\, e^{\nu(r)/2} < 1$ CAPTURED

rafael-lima.pages.dev/talks

OUTLINE

Two papers, *one code*

I

OBSCURA: every collision

A relativistic Monte Carlo of dark-matter capture. Each particle follows the interior geodesic of the star, collides, changes direction and energy — and the code reduces exactly to the optical-depth closure when that extra physics is switched off. What does the closure miss?

de Lima & Rueda, *A Monte Carlo relativistic framework for dark matter capture by neutron stars* (in preparation)

II

Degeneracy before self-gravity

After capture: is the population a classical thermal tracer, a degenerate Fermi gas, or a self-gravitating dark component? Two independent diagnostics — **S** for gravity, **Θ** for statistics — and a map of where their thresholds fall.

de Lima & Rueda, *Degeneracy Before Self-Gravity: Dark Matter Regimes in Neutron Stars* (draft)

THE PREMISE

A neutron star is a *dark-matter detector*

As it crosses the Galaxy, a neutron star sweeps up dark matter: its potential well *focuses* the particles, its matter *stops* them.

halo → focusing → collisions → **capture** → accumulation → regime → heating · collapse · structure

A DEEP WELL

Escape speed at the surface $\sqrt{(2GM/R)}$ = **0.59 c** for 1.4 $M_\odot$ and 11.7 km. Slow halo particles are bent toward the star from a thousand radii away.

A DENSE TARGET

$n_b \approx 5\times10^{38}$ cm^{-3} at the centre: with $\sigma = 10^{-44}$ cm^2 a particle meets $\tau \approx$ **6–9** collisions in one crossing.

AN OLD, COLD STAR

What accumulates can heat it, collapse into a black hole, or change its structure — **if** we know how much is captured and in what state.

I

PART ONE

OBSCURA: *every collision*

A relativistic Monte Carlo that follows each dark-matter particle through the interior metric of the star, collision by collision — and recovers the optical-depth closure when its extra physics is switched off.

the dark-matter background | focusing | the optical-depth closure | the transport | what the closure misses | count laws | cooling



Bullet Cluster, 1E 0657–56 · NASA/CXC/M. Weiss · public domain

1933 – 2006

Matter we only see *through gravity*



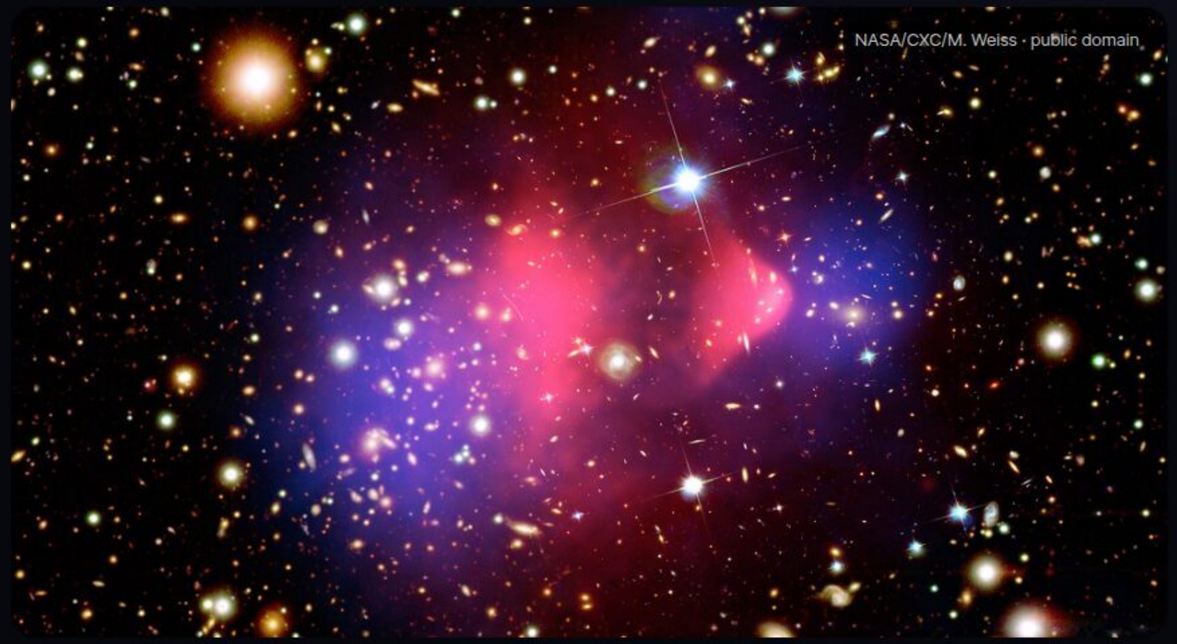
Fritz Zwicky

1933: the Coma cluster moves too fast for its visible mass — *dunkle Materie*



Vera Rubin

1970s: rotation curves of spiral galaxies stay flat far beyond the light



2006, the Bullet Cluster: the lensing mass (blue) is offset from the X-ray gas (pink). Today: about **85%** of the matter, a local density $\rho_\chi \approx 0.4\ \mathrm{GeV\ cm^{-3}}$ — and no idea what it is made of.

Two ways to fill *the halo*

STANDARD HALO · MAXWELL–BOLTZMANN

$$f(v) \propto v^2 \, e^{-v^2/v_0^2}, \; v < v_{\mathrm{esc}}$$

v_0 = 220 km/s, v_{esc} = 544 km/s

The usual WIMP benchmark: particles of GeV to TeV, a few hundred km/s, isotropic. Used for the Maxwell–Boltzmann results of Part II.

RAR FERMIONS · TRUNCATED FERMI–DIRAC

$$f \propto \frac{1 - e^{(E - E_c)/kT}}{e^{(E - \mu)/kT} + 1}, \; E \leq E_c$$

RUFFINI–ARGÜELLES–RUEDA

Self-gravitating fermions in general-relativistic equilibrium: a degenerate core at the Galactic centre and a Boltzmann halo. Rotation curves and the S-stars point to **~60–380 keV**; we take **100 keV**.



FORTY YEARS OF CAPTURE

From the Sun to *neutron stars*

1985
Press & Spergel
capture of WIMPs by the Sun

1987
Gould
the capture-rate formalism

1989
Goldman & Nussinov
WIMPs and neutron stars

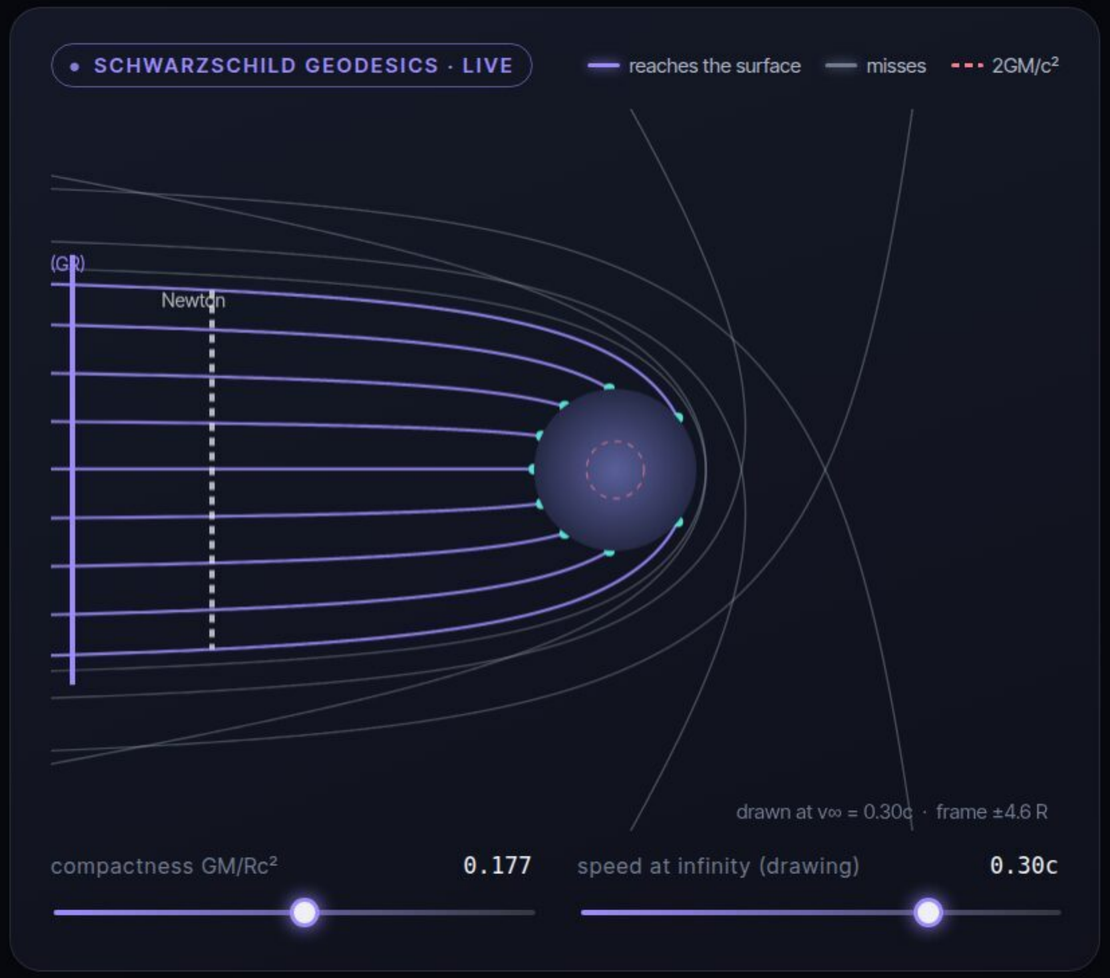
2008–10
Kouvaris; Kouvaris & Tinyakov
heating by annihilation; collapse to a black hole

2017
Baryakhtar et al.
dark kinetic heating

2020
Bell et al.
relativistic capture with Pauli blocking, opacity, multiple scattering

2026
OBSCURA
collision histories through the interior metric

The star looks *a thousand times wider*





$$\frac{b_{\max}}{R} = \frac{1}{\gamma v}\sqrt{(\gamma^2 - 1 + 2C)\,/\,(1 - 2C)}$$

FROM $\mathscr{L} < \mathscr{L}_c(\mathscr{E})$: THE ORBIT MUST REACH R

at this speed `b_max = 2.65 R` · Newton `2.21 R` · GR/Newton `1.198`
· at 220 km/s: `1010 R` , × `1.244` · slow limit 1/√(1−2C) = `1.244`

SUPPLY × PROBABILITY

What reaches the star, and *what stays*

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2 d\Omega^2$$

THE TOV STAR: SLY, 1.40 M⊙, 11.70 km, C = 0.177

$$\frac{d^2\dot{N}_{\rm in}}{d\mathscr{E}\, d\mathscr{L}} = 8\pi^2\, m^3\, f(\mathscr{E})\, \mathscr{L}, \quad \mathscr{L} < \mathscr{L}_c(\mathscr{E})$$

PARTICLES ENTERING THE SURFACE, PER UNIT COORDINATE TIME

$$\dot{N}_{\rm x} = 8\pi^2 m^3 \iint P_{\rm cap}(\mathscr{E}, \mathscr{L})\, f(\mathscr{E})\, \mathscr{L}\, d\mathscr{L}\, d\mathscr{E}$$

THE CAPTURE RATE: THE KERNEL P_{cap} IS WHAT OBSCURA COMPUTES

$\mathscr{E}$ and $\mathscr{L}$ are the energy and angular momentum per unit mass, measured at infinity; between collisions they are **conserved**, and the particle follows the interior geodesic.

CAPTURED

$$\mathscr{E}_{\rm final} = \gamma_{\rm final}\, e^{\nu(r_{\rm coll})/2} < 1$$

after a collision the particle can no longer reach infinity: γ is its Lorentz factor for a static observer at the collision, $e^{\nu/2}$ the lapse there.

THE OPTICAL-DEPTH CLOSURE

A Poisson count and *three frozen choices*

$$P_{\mathrm{cap}} = 1 - e^{-\tau} \sum_{k < N_{\mathrm{min}}} \frac{\tau^k}{k!}$$

THE NUMBER OF COLLISIONS IS POISSON WITH MEAN T

$$\tau = 2 \int_{r_{\mathrm{tp}}}^{R} \frac{e^{(\nu+\lambda)/2}\, \Gamma_{n\chi}}{\sqrt{(\mathcal{E}^2 - V_{\mathrm{eff}}^2)}}\, dr$$

COLLISIONS ALONG THE INCIDENT GEODESIC, ONE CROSSING

The closure of Sirvente & Rueda (submitted): relativistic and covariant. OBSCURA was built to test it, not to replace it — and reduces to it exactly.

1 · FROZEN TRAJECTORY

Collisions change the energy, never the path: every particle keeps the incident geodesic.

2 · DETERMINISTIC LOSS

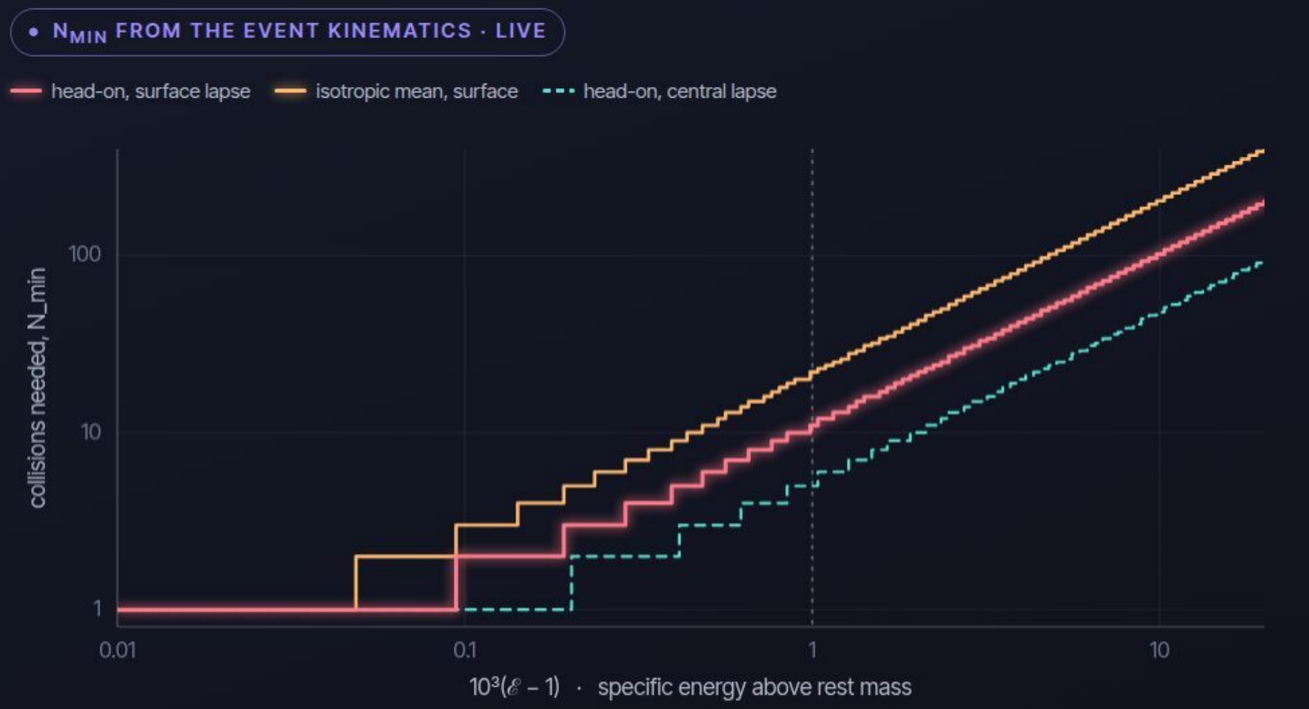
Each collision removes a fixed amount — head-on or the isotropic mean — so capture needs exactly N_{min} of them.

3 · EVALUATED AT THE SURFACE

N_{min} is computed with the lapse at R, wherever the collision actually happens.

100 KEV ON A NEUTRON OF 939 MEV

Capture decided in *a narrow window*



$$\frac{T}{K} = \frac{m_n m_\chi (\gamma + 1)(1 - \cos\theta^*)}{s}$$

KINETIC ENERGY GIVEN TO THE NEUTRON, EXACT

For $m_\chi \ll m_n$ each collision removes a fraction $\sim 2(m_\chi/m_n)(\gamma + 1) \approx 5\times10^{-4}$ at most, while it must lose up to a per cent of it: tens of collisions.

Deeper in the star the lapse is smaller and the same collision removes more of $\mathscr{E}$: N_{min} **halves** at the centre.

Slide the mass to the GeV range: one collision suffices — the WIMP limit.

THE TRANSPORT

One history, *collision by collision*

1. Draw a target optical depth $-\ln U$ and fly along the **interior geodesic** with $(\mathscr{E}, \mathscr{L})$ conserved, accumulating $d\tau = n_b \sigma\, dl$ (step R/4096).
2. At the collision, an **elastic two-body event** on a neutron at rest, isotropic in the centre of mass: energy loss and deflection from the same angles.
3. New γ and direction → new $(\mathscr{E}, \mathscr{L})$: the **orbit is rebuilt**.
4. **Captured** if $\mathscr{E} < 1$; **escaped** if it leaves unbound; otherwise fly again.

$$T = \frac{p^{*2}}{m_n}(1 - \cos\theta^*),\ p^* = \frac{m_n p_\chi}{\sqrt{s}}$$

EXACT, FROM THE INVARIANT t

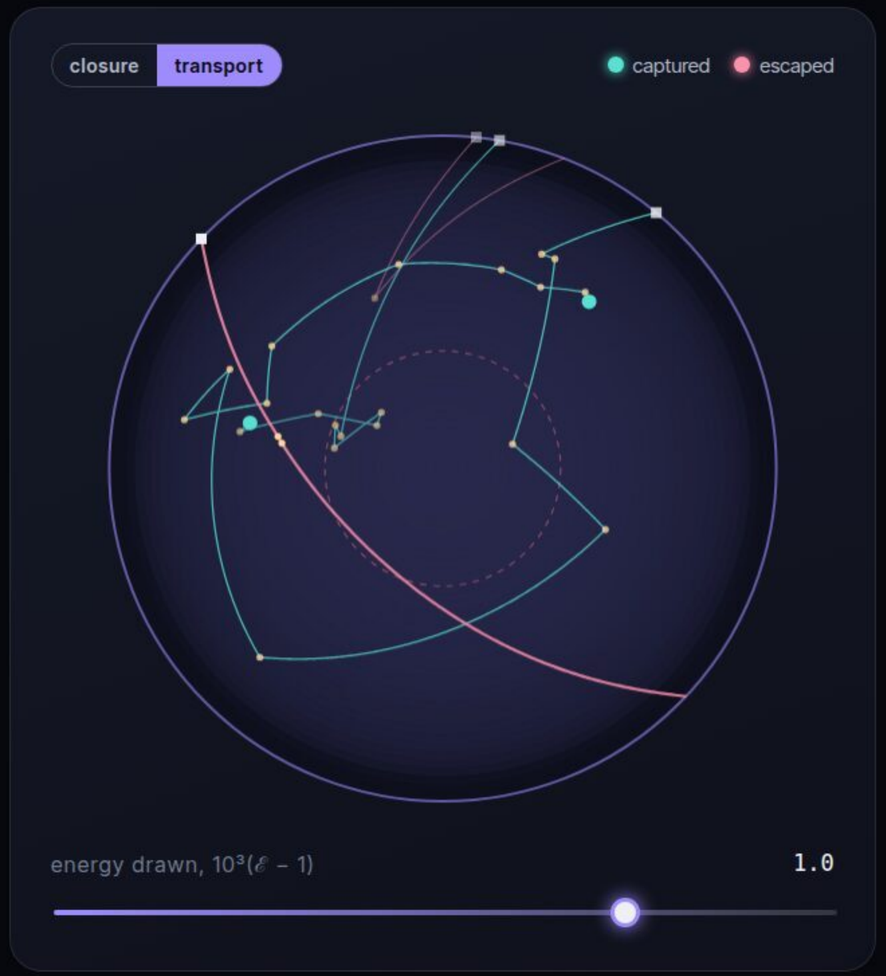
EVERY APPROXIMATION HAS A NAME

`GEO-8` interior tetrad transport · `COL-3` relativistic event · `PAULI-1` blocked rate · `STAT-1` Wilson intervals — each documented, cited in the source, and switchable.

Switch off the release, the sampling and the local lapse, and the Monte Carlo **is** the closure.

THREE MODES, ONE STAR · LIVE

Releasing the trajectory *broadens* the transition





REDUCTION

Switch it off, and it *is* the closure

SWITCH OFF	RECOVERS
trajectory update at collisions	the incident geodesic of τ
sampled energy loss	the recurrence behind N_{min}
loss at the collision radius	the cascade at the surface lapse
histories beyond one crossing	the single-crossing convention
neutrons at rest in the rate	not switched: up to 0.035 in P_{cap}

N_{min}

every capture of the realised closure happens at exactly N_{min}

$0.2_{\%}$

mean count vs $\sigma \int n_b dl$ by an independent quadrature

2.8σ

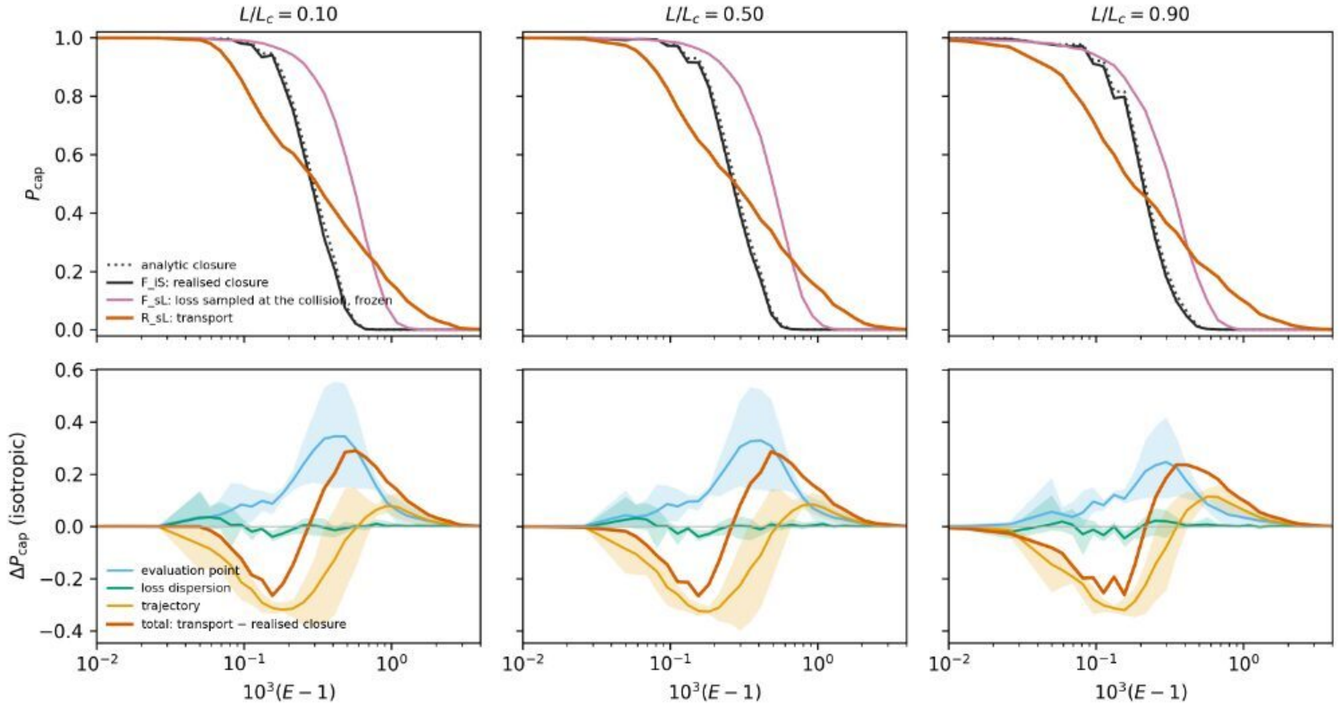
largest deviation of P_{cap} from the Poisson tail, 123 points × 5000 histories

10^{-10}

a sampled event vs an independent closed form; head-on = one step of the recurrence

THE DECOMPOSITION · $2^3 \times 2$ CONTROLS, ORDER-AVERAGED

Two effects of *opposite sign*



SWITCH	MEAN ΔP_{CAP}
• evaluation point	+0.09 … +0.14
• loss dispersion	$\lvert\cdot\rvert$ < 0.01
• trajectory	−0.09 … −0.11
total (signed mean)	+0.00 … +0.03
total, RMS over the transition	0.16 … 0.17

They **nearly cancel in the mean** but not in shape: the first shifts the transition up, the second broadens it. Single energies differ by up to **±0.29**.

An attribution with interactions, not three independent corrections.

Benchmark star, 100 keV, $\sigma = 10^{-44}$ cm^2, 5000 histories per point, no blocking.

MECHANISM · WHAT THE FIRST COLLISION DOES

Half the particles *turn back*

48–53%

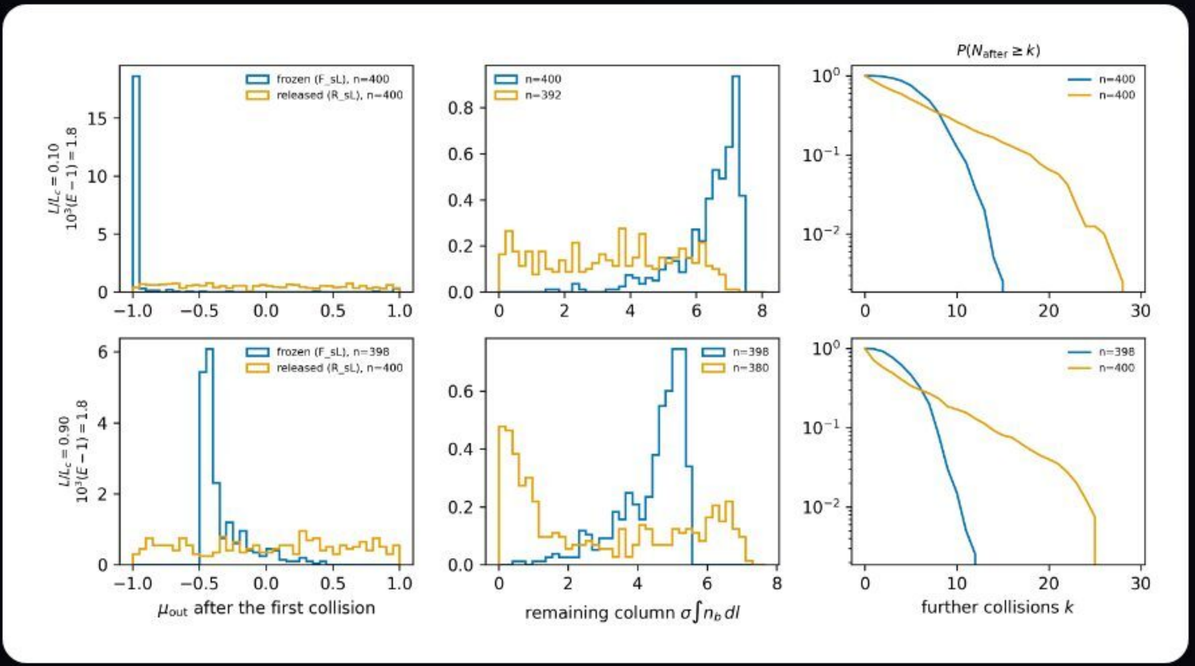
leave the first collision moving **outward** — frozen chord: 2–8%

≈ 3.1

mean column left to the surface, spread over 0.2–6.6 (10–90%) — frozen: 4.4–6.5, narrow

16–28%

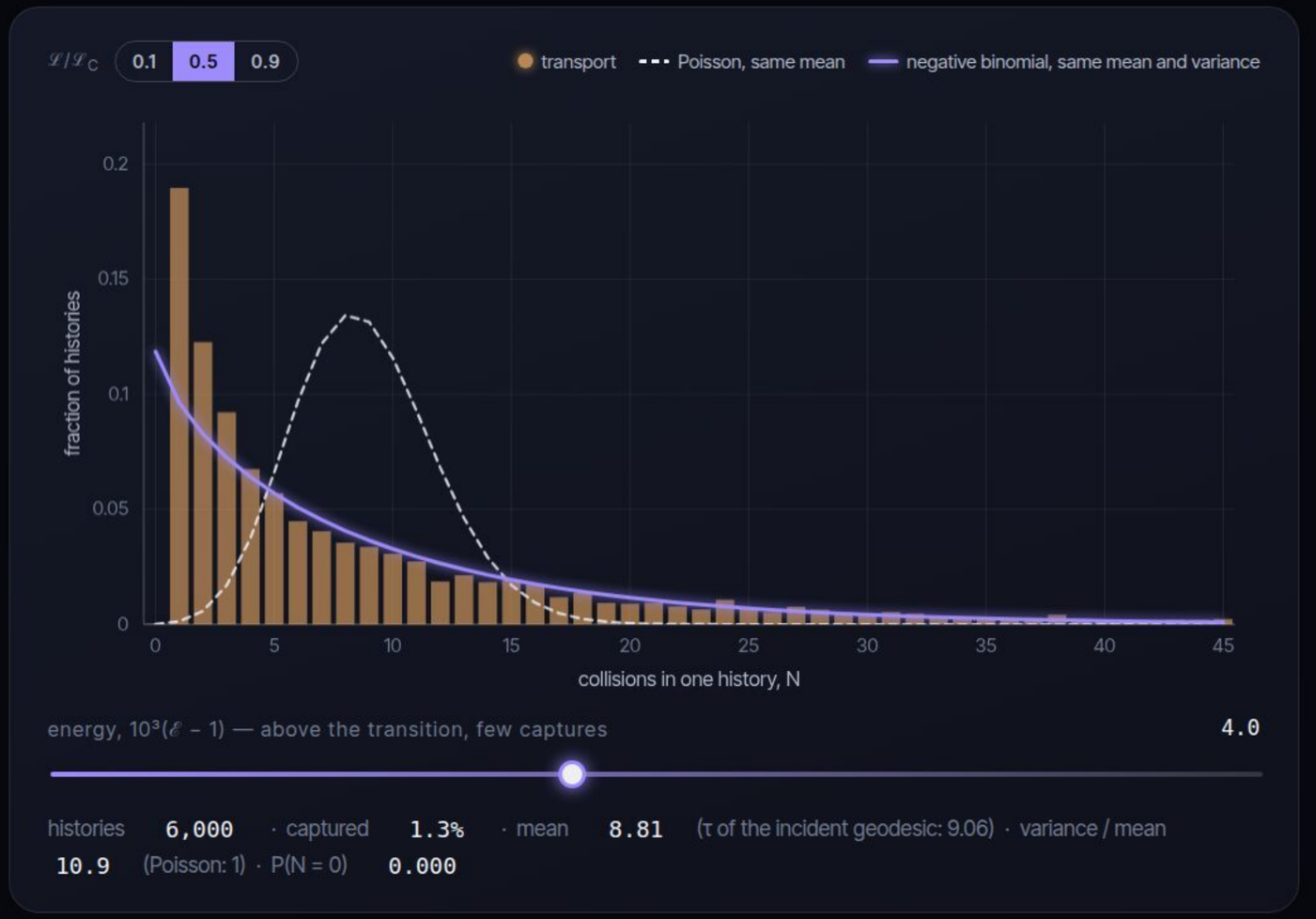
never collide again — frozen: at most 3%



Frozen (F_sL) vs released (R_sL) after the first collision, at $10^3(\mathscr{E} - 1) \approx 1.8$, for $\mathscr{L}/\mathscr{L}_c = 0.1$ and 0.9.

HOW MANY COLLISIONS · LIVE

A count that is *not Poisson*



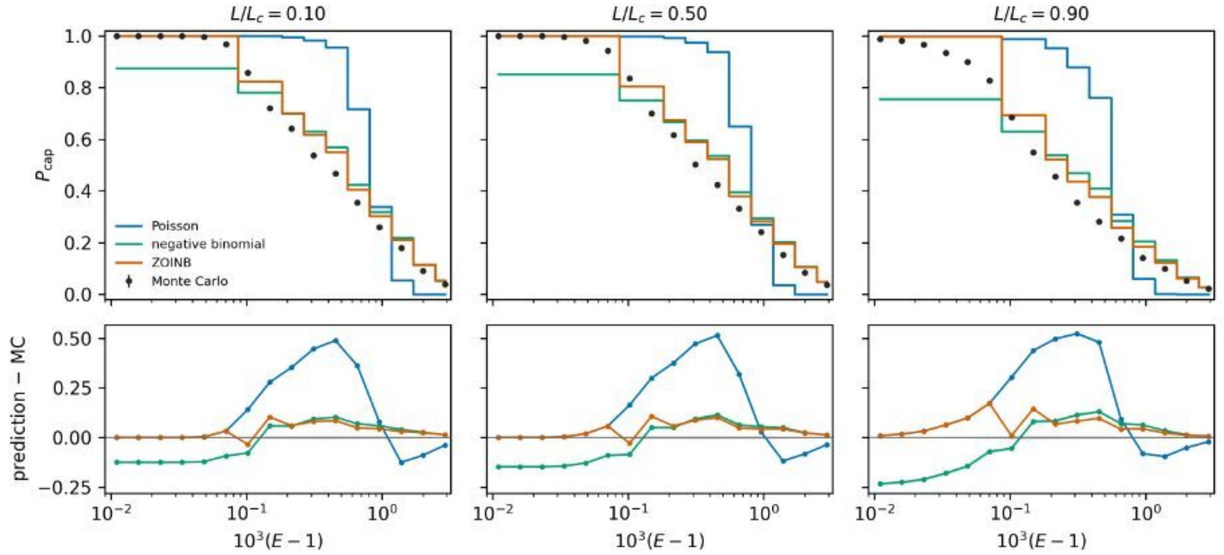
Above the transition almost nothing is captured, so each count is complete: the history ends when the particle leaves. The closure assumes Poisson with the mean τ. The **mean** is right; the **shape** is not.

$$P_{\mathrm{NB}}(k; r, p),\ r = \frac{m^2}{v - m}, p = \frac{m}{v}$$

POISSON COUNT WITH A GAMMA-DISTRIBUTED T

In the paper: variance **8–13 times** the mean. This is a random walk out of an optically thick sphere, not a straight chord.

Better laws, *honest residuals*



MEAN |PREDICTION − MC|	OUR SLY	BENCHMARK
Poisson, measured mean	0.15–0.19	0.16–0.18
negative binomial	0.08–0.11	0.057–0.064
zero-and-one-inflated NB	0.035–0.057	0.044–0.064

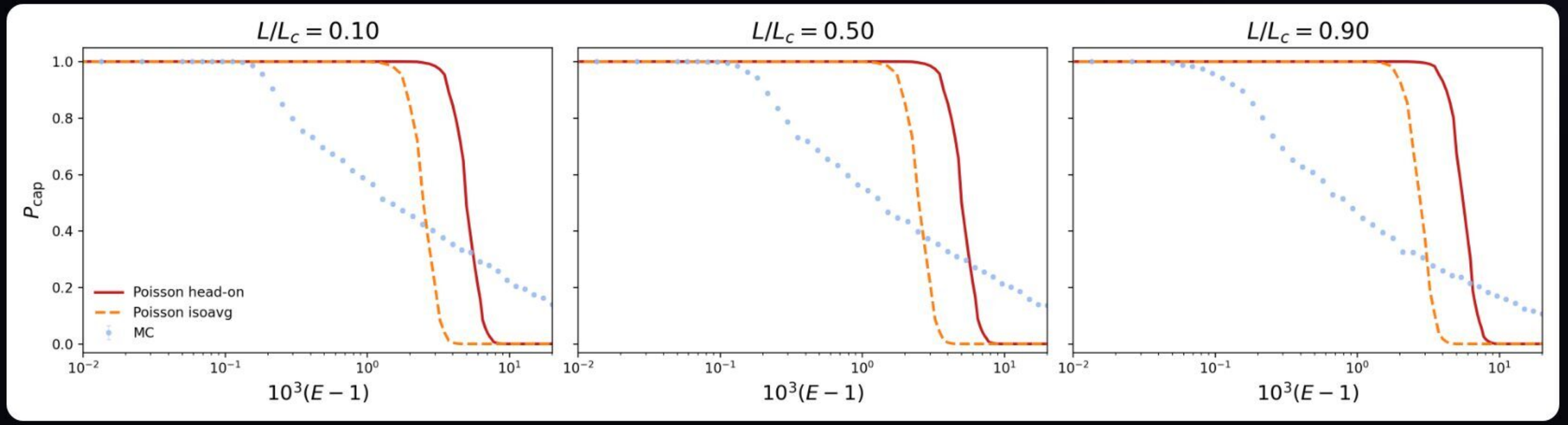
Calibration and test share **no histories**. The zero-and-one-inflated form removes the spurious zero-collision mass of the plain NB.

Residuals stay 6–23 times the combined Monte Carlo and calibration errors: closed forms that help, tested at fixed parameters — not yet a transferable law.

SLy 1.40 M☉, 11.70 km; calibrated above the transition on independent runs, tested inside it.

A MORE COMPACT STAR · SLY AT MAXIMUM MASS, 2.05 $M_\odot$

Broadening is *not a detail*



Points: the transport. Red: closure with head-on N_{min}; orange dashed: isotropic-mean N_{min}. 100 keV, $\sigma = 10^{-44}$ cm², no blocking.

Optical depth of the closure: $\tau \approx$ **25–27**, three times the canonical star.

The transport already falls below 0.95 at $10^3(\mathscr{E} - 1) = 0.1$–$0.2$, and keeps **11–14%** at 20, where both closures vanish.

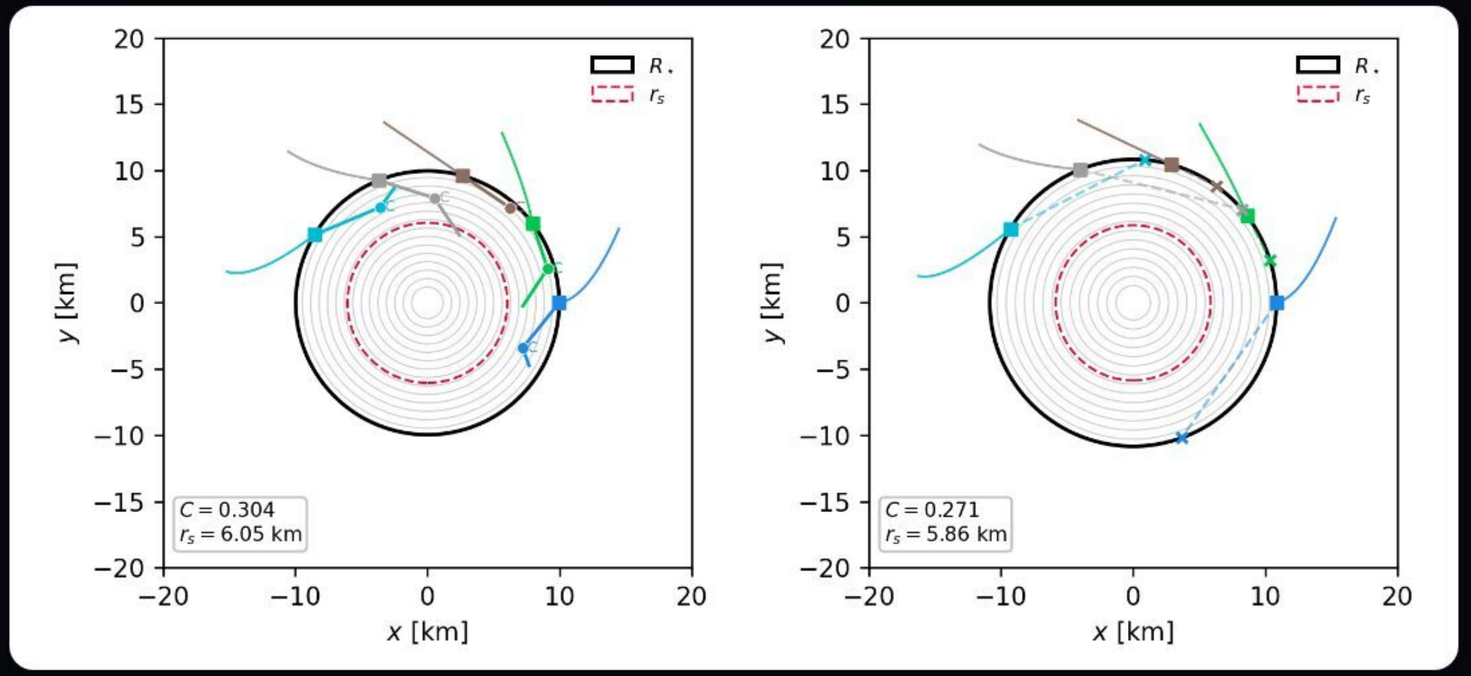
Mean |difference| from the isotropic-mean closure: **0.28–0.29**; single energies up to **0.6–0.7**.

SAME FIVE PARTICLES · DIFFERENT STARS

The equation of state *decides the fate*

SLY · 2.05 M⊙, 9.96 KM · 5/5 CAPTURED

SQM3 · 1.98 M⊙, 10.84 KM · 0/5 CAPTURED



Identical $\mathscr{E}$, $\mathscr{L}$, speed and impact parameter in both panels; σ = 10^{-44} cm^2. Squares: entry; dots: collisions; crosses: escape.

Five incident particles, the **same** phase-space points, sent into the maximum-compactness stars of a hadronic (SLy) and a strange-quark (SQM3) equation of state.

The same orbit crosses a different column and collides at a different depth: **five captures against none**.

Selected cases, not an ensemble average — an illustration of where the EOS dependence of P_{cap} comes from.

BEYOND THE BENCHMARK

Pauli blocking and *cooling*

PAULI BLOCKING · IMPLEMENTED, NOT YET IN THE CAMPAIGNS

The recoiling neutron must leave the Fermi sea: the blocked optical depth is **7×10^{-5}** of the unblocked one.

But the collisions that do happen are violent: on average **57%** of the kinetic energy, against 2×10^{-4} for a static target.

So a blocked τ can never be paired with a static N_{min}: both must come from the **same blocked kernel**. The hierarchy has to be measured again, not rescaled.

Rate tabulated in ($\ln n_b$, $\ln(\gamma - 1)$) to 10^{-3}, the Fermi-surface shell of width $\sim3\times10^{-4}$ resolved by bisection.

COOLING TO 3KT/2 · AFTER CAPTURE

	10^7 K	10^8 K
collisions to capture	5	5
collisions to the threshold	4494	1374
re-entries into the star	396	191
coordinate time	5.1 s	6.8 s
time spent outside the star	97%	99%

Medians. A first-passage time in a cooling-only model — no heating, no detailed balance: **not** thermal equilibrium.

UNDER THE HOOD

Validated *stage by stage*

CHECK	AGREEMENT
interior transport vs independent orbit quadrature (path, azimuth)	$1–2\times10^{-4}$
P_{cap} with the step refined from $R/4096$ to $R/16384$	$\le 2\times10^{-4}$
exterior periapsis vs analytic turning point	$< 1.4\times10^{-13}$
GR / Newtonian focusing at $v \to 0$	1.244
surface hits: quadrant asymmetry, 10^6 samples	3×10^{-3}
CPU vs CUDA, 60 cases × 10^5 histories	4.5×10^{-3}
population equation vs $N = C_X t$	10^{-16}
Fermi EOS: Γ_{eff} at the two ends	1.6667 · 1.3333

OBSCURA

SIMULATING THE INVISIBLE

DARK MATTER | NEUTRON STARS | RELATIVITY

3

independent implementations of the N_{min} recurrence, same integers

425

assertions in 26 test suites

C++17 with OpenMP; the same engine compiled to WebAssembly for an interactive lab.

PART I · IN SHORT

What the closure *misses*

- The Monte Carlo reduces **exactly** to the Poisson closure when its released ingredients are switched off.
- Where the loss is evaluated and whether the trajectory is released move P_{cap} by **±0.1** each, in opposite directions: right on average, **0.16–0.18 RMS** off in shape.
- Collision counts are strongly **overdispersed**; negative-binomial laws calibrated out of sample do much better than Poisson.
- Pauli blocking changes rate **and** kernel together: it must be measured, not rescaled.

> *A correct mean number of collisions is not enough for a Poisson closure.*
>
> the one-line summary of Part I

II

PART TWO

Degeneracy before *self-gravity*

A captured population can be far too light to change the star — and still too dense to be a classical gas. Two diagnostics, and a map of where their thresholds fall.

tracer, Fermi gas or dark core? | S and Θ | the fiducial population | the regime map | what comes next



Neutron star over Manhattan · NASA's Goddard Space Flight Center · public domain

THE QUESTION

After capture, *what is it?*

capture supplies $N_\chi(t)$ → **which regime?** → the right next model

A CLASSICAL TRACER

The usual assumption: the particles thermalise into a Boltzmann cloud in the fixed potential of the star. Right for a dilute gas.

$$n \propto e^{-m\Delta\Phi/kT}$$

A DEGENERATE TRACER

Fermions in a deep well can reach a phase-space density where **Pauli** matters — while still too light to change the gravity.

$$E_F \gtrsim kT$$

A DARK COMPONENT

Dark-matter admixed neutron stars: the dark fluid takes part in hydrostatic balance and shifts the mass–radius relation.

two-fluid TOV

TWO INDEPENDENT DIAGNOSTICS

One for gravity, *one for statistics*

$$S = \frac{M_\chi(r_\chi)}{M_b(r_\chi)}$$

SELF-GRAVITY · INSIDE THE REGION THE DARK MATTER OCCUPIES

S	REGIME	OMITTED GRAVITY S/(1 + S)
$< 10^{-3}$	tracer	$< 10^{-3}$
$10^{-3} – 10^{-1}$	transitional	up to 9%
$\geq 10^{-1}$	self-gravitating	≥ 9%

Local, not the global $M_\chi/M_\star$: $r_\chi = r_{90}$ holds 90% of the particles.

$$\Theta = \frac{T}{T_F} = \frac{k_B T}{E_F}$$

CLASSICALITY · THERMAL ENERGY OVER FERMI ENERGY

Θ	REGIME	FERMI SHARE 1/(1 + Θ)
> 10	thermal	< 0.09
0.1 – 10	transitional	0.09 – 0.91
< 0.1	degenerate	> 0.91

There is no general reason for the two sets of thresholds to **coincide**.

THE TWO CLOSURES

A Boltzmann cloud, and *the Fermi limit*

$$n_\chi(r) = n_0 \exp\left[-\frac{m_\chi \Delta\Phi(r)}{k_B T}\right]$$

THERMAL TRACER IN THE FIXED POTENTIAL OF THE STAR

$$N_\chi = 4\pi \int_0^R n_\chi r^2 \sqrt{g_{rr}}\, dr$$

FIXES n_0 · r_{90} HOLDS 0.9 N_χ

$$p_F = \hbar\,(3\pi^2 n_\chi)^{1/3},\ x_F = \frac{p_F}{m_\chi c}$$

SPIN ½, g = 2

$$E_F = m_\chi c^2\,(\sqrt{(1 + x_F^2)} - 1)$$

$P \propto \rho^{5/3}$ for $x_F \ll 1$ · $P \propto \rho^{4/3}$ for $x_F \gg 1$

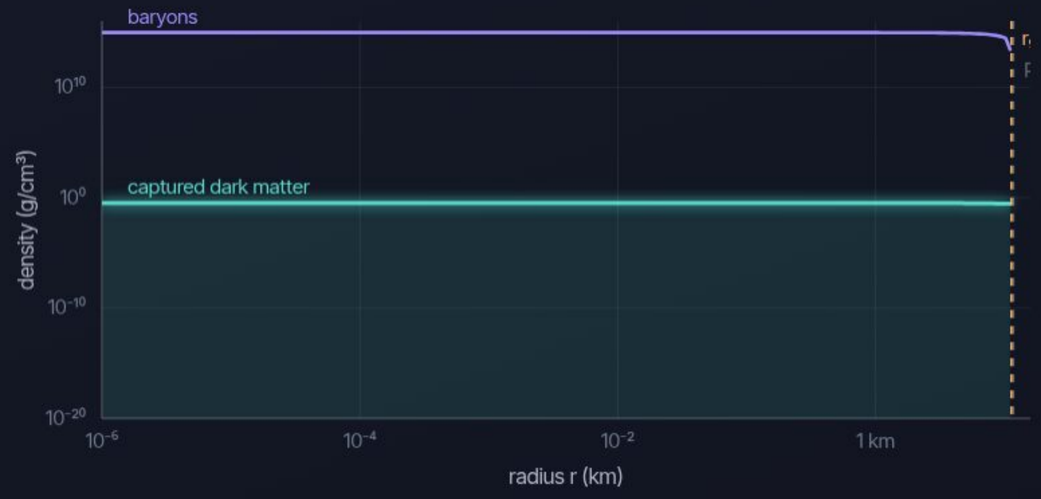
The cloud has no edge: r_{90} ignores the dilute tail and measures where the particles actually are.

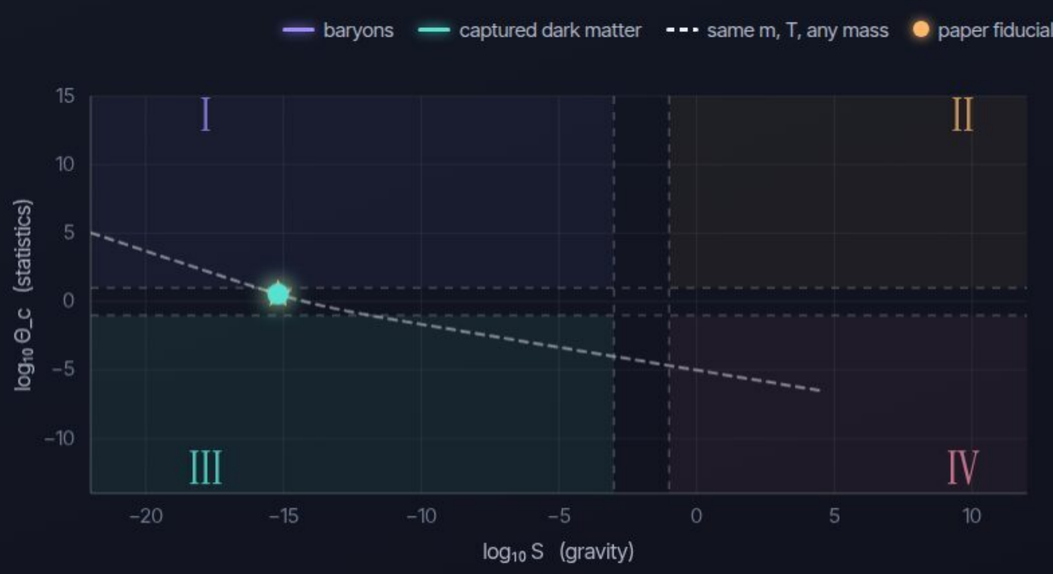
Θ is a **diagnostic ratio**, not a finite-temperature chemical potential: it says when a Fermi–Dirac treatment is needed.

THE TWO DIAGNOSTICS · LIVE

Move a cloud through *the plane*

PRESETS: **paper fiducial · 100 keV, 10^9 K** | 1 GeV WIMP, 10^5 K | 1 keV, 10^6 K | 10^6 GeV, 10^5 K





Dashed lines: the thresholds $S = 10^{-3}, 10^{-1}$ and $\Theta_c = 10, 0.1$. The bands between them are transitional.

I Boltzmann tracer $S < 10^{-3}, \Theta_c > 10$

II Boltzmann, self-gravitating $S \geq 10^{-1}, \Theta_c > 10$

III Fermi tracer $S < 10^{-3}, \Theta_c < 0.1$

IV Fermi, self-gravitating $S \geq 10^{-1}, \Theta_c < 0.1$

particle mass m_χ **100 keV** | core temperature **1.0×10^9 K** | captured mass M_χ **2.36×10^{18} g**

r_{90} **11.3 km** · central density **0.328 g/cm³** · x_F **0.748** · E_F **24.9 keV** vs kT **86.2 keV** · S **6.37×10^{-16}** **TRACER** · Θ_c **3.46** **TRANSITIONAL**

· on this line $\Theta_c = 1$ at **$S \approx 10^{-14}$**

Pauli comes *first* for light particles

A compact cloud sits in the harmonic core, $\Delta\Phi = (2\pi/3)G\rho_c r^2$. Its size cancels between S and the central density:

$$n_0 \approx 4.6\, S\, \frac{\rho_c}{m_\chi}$$

INDEPENDENT OF T

$$\Theta_c \propto T\, m_\chi^{5/2}\, S^{-2/3} \Rightarrow S(\Theta_c = 1) \propto \frac{m_\chi^{5/2}\, T^{3/2}}{\rho_c}$$

NON-RELATIVISTIC FERMI GAS

Light particles are many per gram: they fill phase space long before they weigh anything.

PARTICLE, CORE	S AT $\Theta_c = 1$
1 keV, 10^6 K	10^{-24}
100 keV, 10^9 K	10^{-14}
1 GeV, 10^5 K	10^{-11}
1 GeV, 10^7 K	10^{-8}
1 TeV, 10^5 K	10^{-4}

Self-gravity starts at $\mathbf{S \sim 10^{-3}}$. For anything lighter than about a TeV, the gas turns **degenerate first**.

Full thermal profile in the SLy star, as in the previous panel.

THE PIPELINE

From the star to *the diagnostics*

baryonic TOV star → $b_{max}(V)$ → Γ_{inc} → f_{cap} (Monte Carlo) → C_χ → $N_\chi(t)$ → $\rho_\chi(r)$ → (S, Θ)

$$\Gamma_{inc} = n_\chi \pi \int b_{max}^2 v\, f(v)\, dv$$

FOCUSED FLUX ONTO THE SURFACE

$$C_\chi = \Gamma_{inc}\, f_{cap}$$

f_{cap}: CAPTURED OVER SAMPLED

$$\frac{dN_\chi}{dt} = C_\chi - C_A N_\chi^2 - C_E N_\chi$$

ANNIHILATION, EVAPORATION: OFF IN THE FIDUCIAL

The order matters: capture supplies the particle budget, S tests whether the fixed gravitational background is adequate, Θ tests whether the classical closure is.

THE FIDUCIAL POPULATION · AFTER 10^9 YEARS

Gravitationally invisible, *not quite classical*

	MAXWELL–BOLTZMANN	RAR–FD
incidence rate Γ_{inc}	4.36×10^{29} s^{-1}	2.33×10^{29} s^{-1}
capture fraction f_{cap}	0.962	0.959
captured particles N_χ	1.32×10^{46}	7.06×10^{45}
captured mass M_χ	2.36×10^{18} g	1.26×10^{18} g
cloud radius	11.3 km (r_{90})	11.7 km (support)
self-gravity S	6.4×10^{-16}	3.8×10^{-16}
classicality Θ_c	3.46	5.07

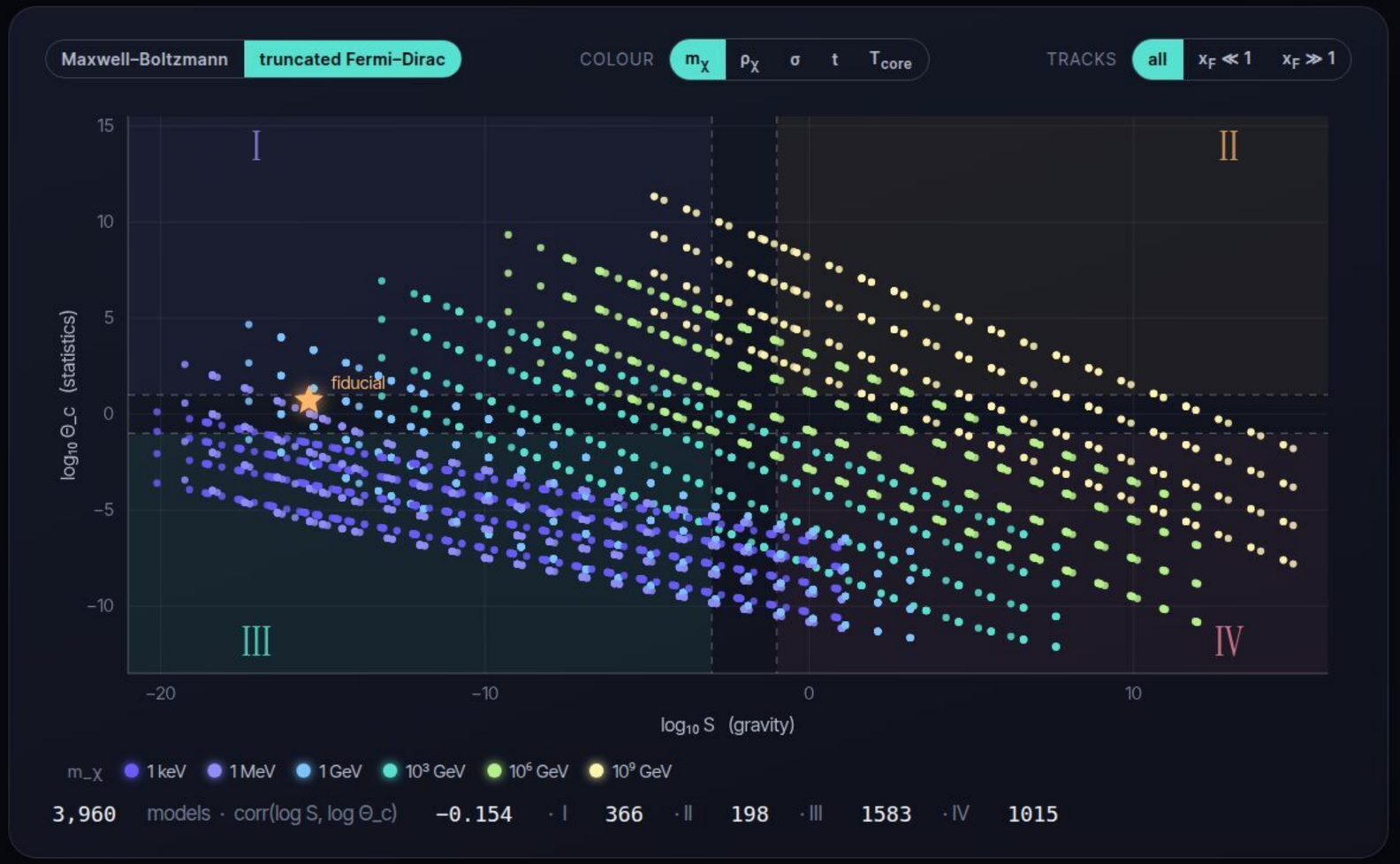
Fiducial: $m_\chi = 100$ keV, $\sigma = 10^{-44}$ cm^2, $\rho_\chi = 0.4$ GeV cm^{-3}, $T_{core} = 10^9$ K, $t = 10^9$ yr, SLy 1.40 $M_\odot$.

$M_\chi/M_\star \approx 10^{-15}$: no mass–radius shift to look for. Yet $\Theta_c \approx 3$–5 is in the **transitional** band: corrections of order one to Boltzmann.

Changing the halo from MB to RAR–FD changes the **supply** by 47% and the microscopic capture by 0.3%.

THE REGIME MAP · EVERY MODEL OF THE SURVEY

The two boundaries *do not coincide*



I

BOLTZMANN TRACER

the usual assumption

II

BOLTZMANN, SELF-GRAVITATING

heavy, classical, massive

III

FERMI TRACER

degenerate, gravitationally passive

IV

FERMI, SELF-GRAVITATING

toward a dark core

READING THE MAP

Two slopes, *one Fermi energy*

Hold m_χ, σ, t, T fixed and change ρ_χ: the profile keeps its shape, $S \propto n_0 \propto N_\chi$, and $p_F \propto n_0^{1/3}$:

$$\Theta_c \propto S^{-2/3}$$

$x_F \ll 1 \cdot E_F \approx p_F^2/2m$

$$\Theta_c \propto S^{-1/3}$$

$x_F \gg 1 \cdot E_F \approx p_F c$

RAR–FD SURVEY	TRACKS	POINTS	MEDIAN SLOPE
non-relativistic family	352	2112	−0.667
relativistic family	213	1278	−0.334

THE MASS TREND

Heavier particles are less degenerate: $d \log \Theta_c / d \log m_\chi \approx$ **0.49** (MB) and **0.42** (RAR–FD). The degenerate quadrants hold **85–89%** of the lightest models and **21–28%** of the heaviest.

NOT A CHANGE OF PARTICLE

III → IV is the same Fermi gas becoming heavier where it sits: the growth of S, not of the particle mass.

MB vs RAR–FD: corr($\log S$, $\log \Theta_c$) = −0.652 vs −0.154.

WHAT COMES NEXT

The right model *for each quadrant*

I

$S \ll 1,\ \Theta \gg 1$

a Boltzmann cloud in the fixed star — the standard assumption, justified

II

$S \gtrsim 0.1,\ \Theta \gg 1$

classical, but it shapes its own potential

III

$S \ll 1,\ \Theta \lesssim 1$

a capture-normalised **Fermi tracer** in the fixed star — finite-T if $\Theta \sim 1$–10

IV

$S \gtrsim 0.1,\ \Theta \ll 1$

only here: a **two-fluid TOV**, the dark-matter admixed star

Capture calculations and admixed-star models are not interchangeable endpoints. A small $M_\chi/M_\star$ or S says the structure is unchanged; it does **not** justify a Boltzmann profile.

NOT YET INCLUDED

Annihilation heating, collapse and black-hole formation, rotation, magnetic fields, and a self-consistent time evolution of the captured Fermi gas. Θ is a diagnostic, not a finite-temperature solution.

CONCLUSIONS

Capture, *then character*

- **OBSCURA** follows dark-matter collision histories through the interior metric and reduces exactly to the optical-depth closure.
- At 100 keV, the frozen trajectory and the surface evaluation each move P_{cap} by **~0.1**, in opposite directions; the count of collisions is **overdispersed**.
- After capture, **S** and **Θ** answer different questions: the fiducial population is an extreme tracer ($S \sim 10^{-16}$) in the transitional band ($\Theta_c \approx 3$–5).
- For light particles **degeneracy comes before self-gravity**: the thermalised-WIMP assumption has to be tested in (S, Θ), not by $M_\chi/M_\star$.

NEXT STEPS

- Pauli-blocked capture campaigns with the consistent blocked kernel.
- Capture rates over the full incident phase space, RAR halo included.
- Thermalisation with heating and detailed balance, then a finite-T Fermi tracer.
- Two-fluid structure once S leaves the tracer regime.

The panels in this seminar run in the browser, on the same physics as OBSCURA: *rafael-lima.pages.dev/talks*.

IMAGE CREDITS

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Paper figures — de Lima & Rueda, *A Monte Carlo relativistic framework for dark matter capture by neutron stars* (in preparation); de Lima & Rueda, *Degeneracy Before Self-Gravity: Dark Matter Regimes in Neutron Stars* (draft)

Regime-map panel — the survey data of the second paper, from the OBSCURA pipeline

Neutron star — SLy equation of state (Douchin & Haensel 2001) in the piecewise-polytropic fit of Read, Lackey, Owen & Friedman, PRD 79, 124032 (2009)

OBSCURA logo — OBSCURA project

THANK YOU

Questions?

Slides and live panels, on the same physics as OBSCURA, at *rafael-lima.pages.dev/talks*

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OBSCURA

SIMULATING THE INVISIBLE